

A Practitioners Toolkit on Valuation

Part IV: Valuing Sales, Costs, and Cash Flows

Frans de Roon, Joy van der Veer

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1 Introduction

A common approach in company valuation is to make a projection of future free cash flows and discount these at the cost-of-capital associated with these cash flows. In a Capital Asset Pricing Model (CAPM)-framework, this usually means that the free cash flows are discounted based on their asset beta or unlevered beta, β_A , from which we obtain the unlevered discount rate:

$$k_U = Rf + \beta_A E[Rm - Rf]. \quad (1)$$

The asset or cash flow beta, β_A , is often derived from comparable listed firms, whose equity beta, β_E , as derived from stock market data, is unlevered according to the firms' financing policy. This way of estimating the beta associated with the cash flows is referred to as a "top-down" approach.¹

In this paper we show how the beta associated with the cash flow can alternatively be derived using a "bottom-up" approach, in which we start from the inherent (market) risk that is present in a company's sales and then reconstruct the risk that is present in the cash flows. Just like deriving the company's asset beta from comparable firms requires stock market data and capital structure knowledge to estimate the equity beta and then infer the unlevered asset beta, deriving the company's asset beta from its sales and

¹ We value cash flows at a cost-of-capital that reflects their inherent risk, without any Debt/Equity effect. This means we work in an Adjusted Present Value framework, and do not use the Weighted Average Cost of Capital.

costs, requires useful data on revenues (sales) and its cost structure.

We will first show how the Certainty Equivalent Valuation method provides a useful framework for valuing cash flows, based on cash flow characteristics such as standard deviation and correlation with the stock market. We will also show how the risk in the cash flow *growth* (i.e., the percentage *change*, in the cash flow, either positive or negative) is different from the risk in the cash flow *return* (i.e., the percentage change in the value of a claim on the cash flow). This difference in growth versus return risk applies both to cash flow volatility (i.e., standard deviation) and cash flow beta. Subsequently, we will show how this translates into the risk of the different elements of the cash flow: sales, variable costs, and fixed costs. Throughout we will use the CAPM as our asset pricing model, but the results shown here are easily generalized to other asset pricing models.

2 Risk-adjusted discounting and Certainty Equivalent Valuation

2.1 The one-period case

We start with the one-period case, where there is an uncertain cash flow at time 1, CF_1 , to be valued at time 0. In valuing a single future expected cash flow, $E[CF_1]$, a common approach - starting from the CAPM - is the risk-adjusted discounted value of that cash flow:

$$V_0 = \frac{E[CF_1]}{1 + Rf + \beta_A E[Rm - Rf]}. \quad (2)$$

If this is a cash flow that occurs to a 100%-equity financed company (i.e., there is no leverage) then β_A can be obtained directly from the equity returns R_E , i.e., $\beta_A = \beta_E = Cov[R_E, R_M]/Var[R_M]$. Manipulating (1), we can also write it as the Certainty Equivalent Value of the expected cash flow:²

$$V_0 = \frac{E[CF_1] - \lambda_m Cov[CF, Rm]}{1 + Rf}, \quad (3)$$

$$\lambda_m = \frac{E[Rm - Rf]}{Var[Rm]}. \quad (4)$$

Here, λ_m is the market price of risk, which is the excess return (in excess of the risk free rate Rf) that investors require for bearing market risk, $Var[Rm]$. Whereas the valuation in (2) takes the expected cash flow as a given and adjusts the discount rate in the denominator for the risk inherent in the cash flow, Equation (3) adjusts the expected cash flow in the numerator with a discount, such that it becomes a *certainty equivalent* and can be discounted at the risk free rate. Although both valuation approaches are equivalent and lead to the same result, an important difference is that in (2) we need the covariance between returns and the market return to determine β_A , whereas in (3) we need the covariance between the cash flow and the market return, $Cov[CF, Rm]$.

To make the difference between the two approaches in (2) and (3) more explicit, and to also prepare for valuing multi-period cash flows, we follow Fama (1977), and assume that (expected) cash flows evolve according to

$$CF_1 = E_0[CF_1](1 + \varepsilon_1). \quad (5)$$

We will generalize this shortly, but for now Equation (5) tells us that the realized cash flow at time 1 is the expectation of that same cash flow at time 0, $E_0[CF_1]$ multiplied by an unexpected (mean zero) growth term ε_1 . Thus, ε_1 is the unexpected percentage change in the cash flow. Substituting (5) into (3) gives

$$V_0 = \frac{E_0[CF_1] - \lambda_m Cov[\varepsilon, Rm] E_0[CF_1]}{1 + Rf}$$

² See, e.g. Fama (1977), Weston & Copeland (1986), and Copeland & Antikarov (2003).

$$V_0 = \frac{E_0[CF_1](1 - \lambda_m \sigma_{\varepsilon m})}{1 + Rf}, \quad (6)$$

with $\sigma_{\varepsilon m} = Cov[\varepsilon, Rm]$. It is also important to note that in (6)

$$\begin{aligned} \lambda_m \sigma_{\varepsilon m} &= \frac{Cov[\varepsilon, Rm]}{Var[Rm]} E[Rm - Rf] \\ &= \beta_{CFg} E[Rm - Rf]. \end{aligned} \quad (7)$$

The two betas, or covariances, that are used in the risk-adjusted discounting in (2) versus the certainty equivalent in (3) and (7), i.e., β_A versus β_{CFg} are different in nature. The beta in (2), β_A , also known as the unlevered beta, is the beta on a company that generates the cash flow CF and that is financed with only equity. β_A is then the regression slope coefficient from regressing the returns of this company's stock returns on the market returns. When the company does have financial leverage, one normally estimates β_A by unlevering the equity beta β_E . The beta in (7) on the other hand reflects the unexpected growth in the cash flow, ε , and is the regression slope coefficient of the (unexpected) cash flow growth, or change, on the market returns. Using Equations (2) - (7) above, it is straightforward to show (see the Internet Appendix) that the two betas are related as follows:³

$$\beta_A = \beta_{CFg} \left(\frac{1 + Rf}{1 - \lambda_m \sigma_{\varepsilon m}} \right). \quad (8)$$

At this point, it useful to illustrate this with a simple example. We are interested in valuing an expected cash flow of € 3,000, which is expected to be received one year from now. The uncertainty in this cash flow, i.e., the unexpected (positive or negative) growth as a percentage of this expectation is 40%. Exhibit 1 summarizes the other input data needed.

³ And note that combining with (7) this can also be written as

$$\beta_A = \beta_{CFg} \left(\frac{1 + Rf}{1 - \beta_{CFg} E[Rm - Rf]} \right).$$

Exhibit 1: Input data	
E[Rm-Rf]	5.0%
sigma_m	15%
Rf	4.0%
correl(e,m)	0.40
sigma_e	40%
CF	3,000

From the expected market risk premium and the market volatility, we obtain the market price of risk: $\lambda_m = E[Rm - Rf] / \sigma_m^2 = 5\% / 15\%^2 = 2.22$. The covariance of the unexpected cash flow growth with the market return, is $Cov[\varepsilon, Rm] = \sigma_\varepsilon \sigma_m \rho_{\varepsilon m} = 40\% \times 15\% \times 0.40 = 0.024$. Thus, the present value of the expected cash flow is, from (6):

$$V = \frac{3,000 \times (1 - 2.22 \times 0.024)}{1.04} = 2,731.$$

The beta of the cash flow growth follows from the covariance and the variance of the market return as $\beta_{CFg} = 0.024 / 0.15^2 = 1.07$. Using Equation (8) then tells us that the asset beta itself is

$$\beta_A = 1.07 \times \left(\frac{1.04}{1 - 2.22 \times 0.024} \right) = 1.17.$$

This beta, which is also the unlevered beta, gives us subsequently the discount rate for the same cash flow, as in Equation (1): $k_U = 4\% + 1.17 \times 5\% = 9.9\%$. Using this discount rate to value the same cash flow, naturally gives the same result:

$$V = \frac{3,000}{1.099} = 2,731.$$

As a final remark in this section, note that the same relation between the betas in (8) also applies to the standard deviations:

$$\sigma_A = \sigma_{CF} = \sigma_\varepsilon \left(\frac{1 + Rf}{1 - \lambda_m \sigma_{\varepsilon m}} \right). \quad (9)$$

On the right hand side, σ_ε is the standard deviation of the unexpected cash flow growth, which is 20% in the example. The resulting σ_{CF} is

$$\sigma_{CF} = 40\% \times \left(\frac{1.04}{1 - 2.22 \times 0.024} \right) = 43.9\%.$$

This should be interpreted as the return standard deviation of the value of the cash flow. Thus, if the

company would be unlevered, the volatility of the stock returns (σ_E) would be 43.9%.⁴

2.2 The multiperiod case

Before we start applying the results from Section 2.1 to the valuation of the elements of the cash flows (sales and costs), we first proceed by generalizing the certainty equivalent valuation to the multiperiod case. The purpose is to determine the present value of a cash flow that occurs at time t (i.e., t periods ahead). Building on (5), as in Fama (1977), the expected cash flow evolves as

$$E_1[CF_t] = E_0[CF_t](1 + \varepsilon_1), \quad (10)$$

$$E_2[CF_t] = E_1[CF_t](1 + \varepsilon_2), \quad (11)$$

$$\vdots$$

$$E_s[CF_t] = E_{s-1}[CF_t](1 + \varepsilon_s) \quad (12)$$

$$\vdots$$

$$CF_t = E_{t-1}[CF_t](1 + \varepsilon_t). \quad (13)$$

The time-subscripts for the expectations operator E_s highlight that each time we take the expectation of the cash flow CF_t at time s , taking all available information until time s into account.⁵ For the realized cash flow, from the perspective of time zero, this means

$$CF_t = E_0[CF_t] \times (1 + \varepsilon_1) \times (1 + \varepsilon_2) \dots \times (1 + \varepsilon_t).$$

Again, ε_s ($s = 1, 2, \dots, t$) is the per-period unexpected growth in the expected cash flow. As the actual cash flow is only realized in the last period t , it is natural to think of the ε_s 's as the news or updates about the expected cash flow every period, as is most clearly seen from Equations (10)-(13). We assume that the ε_s 's are uncorrelated with each other and have a constant correlation with the market return only in the

⁴ In real option applications we often have the value of the cash flows, not the cash flows themselves, as the underlying value of the option. σ_{CF} , 43.9% in the example, would then be the volatility to use in the option valuation.

⁵ Thus, $E_s[CF_t]$ is the expectation of CF_t conditional on the information that is known at time s .

same period - thus, ε_s is only correlated with the market return Rm_s and not with market returns in any other period. We also assume that ε_s has expectation zero and the same standard deviation σ_ε in every period. The CAPM parameters (the riskfree rate, the market risk premium, and the market variance) are also assumed to be constant, implying that λ_m is also constant.

To illustrate the multiperiod case, let's take a two-period example, $t = 2$. Thus, we want to know the value today of $E_0 [CF_2]$. Working backwards from time 2, we have at time 1 that the value of the expected cash flow is

$$\begin{aligned} V_1 &= \frac{E_1 [CF_2] - \lambda_m Cov_1 (CF_2, Rm_2)}{1 + Rf} \\ &= \frac{E_1 [CF_2] - \lambda_m Cov_1 (E_1 [CF_2] (1 + \varepsilon_2), Rm_2)}{1 + Rf} \\ &= \frac{E_1 [CF_2] (1 - \lambda_m \sigma_{\varepsilon m})}{1 + Rf}. \end{aligned} \quad (14)$$

The last step follows from the fact that $E_1 [CF_2]$ is known at time 1 and therefore can be treated as a constant. Similarly, at time 0, the value is

$$V_0 = \frac{E_0 [V_1] - \lambda_m Cov_0 [V_1, Rm_1]}{1 + Rf}. \quad (15)$$

Substituting Equation (14) into (15) and using that $E_1 [CF_2] = E_0 [CF_2] \times (1 + \varepsilon_1)$, it follows directly (see the Appendix) that

$$V_0 = \frac{E_0 [CF_2] \times (1 - \lambda_m \sigma_{\varepsilon m})^2}{(1 + Rf)^2}.$$

In general, the present value of a cash flow occurring at time t is

$$V_0 = \frac{E_0 [CF_t] \times (1 - \lambda_m \sigma_{\varepsilon m})^t}{(1 + Rf)^t}. \quad (16)$$

Thus, Equation (16) shows how the certainty equivalent value, where the nominator adjusts the expected cash flow for risk, can easily be applied in the multiperiod case⁶, and thus in any valuation or capital

budgeting exercise. Of course, as in the one-period case, the present value obtained via (16) is equivalent to the value obtained by discounting the expected cash flow at the risk-adjusted discount rate k_U for t periods.

3 Determining the values and betas of the cash flow elements

3.1 From sales to cash flow valuation

Having established the certainty equivalent valuation method and its equivalence to discounting expected cash flows at the risk-adjusted discount rate, we now want to apply this framework to determine the value, betas, and discount rates that go with each of the elements of the cash flow. Starting from a simple (free) cash flow statement as in Exhibit 2, the idea is that each element in the cash flow has its own risk, and therefore beta and discount rate, and that the cash flow is essentially a portfolio of these different elements. The risk (and beta, and discount rate) that are associated with the cash flow, is therefore the result of the constituting elements.

One advantage in practice of using this approach, is that we do not necessarily need good historical data on cash flows, but we can start from historical data on sales, which are often less noisy. By applying the cost structure of the company we can then derive the risk profile of the cash flow from the risk profile of the sales and the cost structure - i.e., a *bottom-up* approach.⁷

⁶ See also Weston & Copeland (1986).

⁷ In Part V of this toolkit, we will provide a framework for how to derive the risk profile of sales from historical sales data.

Exhibit 2: Cash Flow Statement	
Sales	
- Cost of Goods Sold	
- Selling & General Administrative Expenses	
- Depreciation	
EBIT	
- Tax on EBIT	
NOPAT	
Depreciation	
Gross Cash Flow	
- Change in Working Capital	
- Capital Expenditures	
Free Cash Flow	

In order to illustrate the idea of building the cash flow profile starting from sales and the cost structure, we take a simplified case in which there is no depreciation and no capital expenditures, and where the change in working capital is expected to be zero. Exhibit 3 then guides through an example of how to derive the relevant cash flow information. All other inputs are as in Exhibit 1.

Exhibit 3: Values and Betas of Cash Flow Elements			
	Level	sigma	PV
Sales	10,000	20%	9,359
COGS	20% - 2,000	20%	- 1,872
SGA	- 4,000	0%	- 3,846
EBIT	4,000	40%	3,641
Tax	25% - 1,000	40%	- 910
Cash Flow	3,000	40%	2,731
	beta_g	beta_r	k
Sales	0.53	0.57	6.8%
COGS	0.53	0.57	6.8%
SGA	0.00	0.00	4.0%
EBIT	1.07	1.17	9.9%
Tax	1.07	1.17	9.9%
Cash Flow	1.07	1.17	9.9%

We will focus on the one period-case, where there is an expected level of sales at the end of year 1 of € 10,000, with a standard deviation of 20%. As with the cash flow, this means that the unexpected growth in sales has a standard deviation of 20%. The cost structure is such that there are variable costs, costs of goods sold, which are 20% of sales, and fixed costs, which are € 4,000. The applicable tax rate is 25%. This already allows us to derive the standard deviation of the (unexpected growth of the) cash flow.

If COGS are completely variable, then they have exactly the same volatility as (and are perfectly correlated with) sales. Thus, the standard deviation of COGS, which are € 2,000, is also 20%. We also take the fixed costs, the selling & general administrative expenses, to be truly fixed, implying that there is no uncertainty here, and the standard deviation of SGA must be 0%.⁸ Using this, we can now derive the standard deviation of EBIT, since EBIT is essentially a portfolio of sales and costs:

$$\sigma_{EBITg} \times EBIT = \sigma_{Sg} \times Sales - \sigma_{Vg} \times COGS - \sigma_{Fg} \times SGA,$$

The subscripts V and F stand for variable and fixed costs respectively. For truly variable and fixed costs we have $\sigma_{Vg} = \sigma_{Sg}$ and $\sigma_{Fg} = 0$, so we can also write

$$\sigma_{EBITg} = \left(1 + \frac{Fixed\ Costs}{EBIT}\right) \times \sigma_{Sg}.$$

We add g to the subscripts, to emphasize that we refer to the standard deviation of the unexpected growth, or percentage change, of the various elements. In the example we have:

$$\begin{aligned} \sigma_{EBITg} \times 4,000 &= 20\% \times 10,000 - 20\% \times 2,000 \\ &\quad - 0\% \times 4,000 \\ \sigma_{EBITg} &= 20\% \times 8,000 / 4,000 = 40\%. \end{aligned}$$

Applying the same logic, the taxes are 25% of EBIT, and thus have the same standard deviation as (and are perfectly correlated with) EBIT. The same applies to the cash flow in this example (where cash flow equals NOPAT), which is the remaining 75% of EBIT. Thus, we find that also our cash flow has, in terms of its unexpected growth, a standard deviation of 40%.

Armed with this standard deviation and the correlation of the cash flow (growth) and the market return of 0.40, we can now - using the additional input

⁸ Obviously, fixed costs will hardly ever be really fixed, implying that they will have a standard deviation and a possible correlation with sales. The framework and the example is easily adjusted for this, but we abstract from it here to keep the example tractable.

parameters from Exhibit 1 - determine the present value of the expected cash flow as

$$V = \frac{3,000 \times (1 - 2.22 \times 0.40 \times 0.15 \times 0.40)}{1.04} = 2,731.$$

Also combined with the input variables in Exhibit 1, the standard deviation of 40% implies that the cash flow (growth) beta is $\beta_{CFg} = 0.40 \times 0.40 / 0.15 = 1.07$. For the return-based beta, β_A this means (Equation (8)):

$$\beta_{CF} = 1.07 \times \left(\frac{1.04}{1 - 0.0533} \right) = 1.17.$$

The discount rate for the cash flows is therefore $4\% + 1.17 \times 5\% = 9.9\%$. Valuing the expected cash flow of 3,000 at this discount rate of 9.9%, obviously gives the same present value of € 2,731.

3.2 Operating leverage

Although going from the sales to the cash flows as we did in the previous section suffices to determine the beta and the discount rate of the cash flow, it is useful to have a further look at the cash flow elements in Exhibit 3. In particular, the valuation and beta of each cash flow element individually provide some useful insights.

First, notice that we can think of each element (sales, COGS, SGA, etc.) as a cash flow in itself, that represents a value. This is reflected in the third column, "PV", where each element is valued using the certainty equivalent valuation in Equation (6). Since we know each element's standard deviation, and the correlation with the market is the same for each element (except for SGA, where it is zero), we can apply Equation (6) and value all elements separately. Of course, these values add up in the same way as the elements in the first column do. Thus, just like EBIT is the result of sales minus COGS minus SGA, so is the *value* of EBIT the result of the *value* of sales minus COGS minus SGA. Looking at the total cash flow statement, we thus have that the value of the cash flow, 2,731, is the value of sales minus the value of COGS, SGA and taxes ($9,359 - 1,872 - 3,846 - 910 = 2,029$). Looked upon in this way, the cash flow is really a portfolio of sales and the various cost items.

Second, also in terms of these values, each element has its own return beta, which is different from its growth beta - but it is this return beta that determines the risk-adjusted discount rate of each item. Based on the growth standard deviation of sales of 20%, and the correlation with the market return of 0.40, the return beta of sales is 0.57. This can be interpreted as the inherent business risk of the company (or project). As in the end all value and risk starts with sales, the market-risk, or beta, of sales, represents the starting point of the risk profile of the company. If we would have a claim on sales in this company, i.e., if we could trade a security whose payoff would only depend on the company's sales, it would have a beta of 0.57, and an accompanying cost-of-capital of 6.8%. Going down the cash flow statement though, we see that once we arrive at EBIT (and then cash flow), the return beta has increased to 1.17, and the accompanying cost-of-capital to 9.9%.

Somewhere along the (cash flow statement) lines, the intrinsic business risk, measured as the beta of sales (0.57), has increased to 1.17 - more than double. The key in understanding what has happened is the cost structure. In particular, our cash flow statement shows that variable costs (COGS) are 20% of sales, whereas there is an additional "40%" fixed costs. This induces *operating leverage*. Just like fixed interest payments make the beta of equity, β_E , higher than the cash flow beta, or unlevered beta ($\beta_A = \beta_U$) so do fixed costs make the cash flow beta, β_A higher than the intrinsic "sales" beta, β_S . This is a key insight. A company's intrinsic risk is derived from its sales - how uncertain are the revenues the company generates. Fixed cash outflows, either as costs (operating leverage) or as interest payments (financial leverage) lead to an increased risk profile for the shareholders. This is reflected in higher betas, and higher discount rates.

This is further illustrated in Exhibit 4, where in the top panel the cash flow statement and the accompanying standard deviations and return betas from Exhibit 3 are repeated. This is based on a free cash flow where COGS is 20% of sales, or € 2,000 (expected), and SGA are € 4,000. The bottom panel shows a similar cash flow statement, where this fixed versus variable cost structure is reversed, i.e., COGS are

now 40% (and thus expected to be € 4,000), whereas SGA is now € 2,000. Note that in both panels the expected EBIT and free cash flow are the same. However, because in the bottom panel the fixed costs are lower and the variable costs are higher than in the top panel, the standard deviations and (return) betas in the two cases are very different. With the lower fixed costs, the standard deviation in the bottom panel is only 30%, compared to 40% in the top panel, implying that the operating leverage in the bottom panel is only half the leverage in the top panel. This is in the same way reflected in the two betas, which are 0.87 (bottom panel) versus 1.17 (top panel) respectively.

Exhibit 4: Two cash flows with different operating leverage

	Cash Flow 1		sigma_g	beta_r
Sales		10,000	20%	0.57
COGS	20% -	2,000	20%	0.57
SGA	-	4,000	0%	0.00
EBIT		4,000	40%	1.17
Tax	25% -	1,000	40%	1.17
Cash Flow		3,000	40%	1.17
	Cash Flow 2		sigma_g	beta_r
Sales		10,000	20%	0.57
COGS	40% -	4,000	20%	0.57
SGA	-	2,000	0%	0.00
EBIT		4,000	30%	0.87
Tax	25% -	1,000	30%	0.87
Cash Flow		3,000	30%	0.87

This insight is also important when thinking about value creation. Albeit exaggerated, the analysis in Exhibits 3 and 4 shows that the discount rate of variable costs is 6.8%, whereas the discount rate of fixed costs is 4.0%. This means that each Euro cost savings (valued as a perpetuity) in variable costs, implies an increase in value of € 14.60, whereas, each Euro cost savings in fixed costs implies an increase in value of € 25.00. This provides a framework to analyse the value effects of cost-saving plans, outsourcing, risk management, etc.

4 Conclusions

The purpose of this part of the toolkit on valuation was to illustrate how asset betas (cash flow betas) can be determined starting from the risk profile of the sales of a company (i.e. the standard deviation of

sales and its correlation with stock market returns). Using the risk profile of sales and the cost structure of a company then allows us to derive the asset beta that represents the inherent risk in the free cash flows. This bottom-up approach is an alternative that can be used in practice as an alternative for the top-down approach, where asset betas are derived from equity betas of comparable listed companies, which are subsequently adjusted for the financing policies of these companies.

An important tool in the analysis is the certainty equivalent valuation framework. This framework highlights the difference between a growth beta that is obtained from regressing flows like sales, EBIT and free cash flows on stock market returns, versus a return beta, that is obtained from regressing (equity) returns on stock market returns. Whereas the bottom-up approach starts from betas and standard deviations in sales or cash flow changes, or growth, the top-down approach starts from equity return betas. The certainty equivalent valuation framework allows us to link these two types of betas and standard deviations to each other.

As the starting point of the bottom-up approach is the risk profile of sales, the next step is to determine this risk profile. The next part of this toolkit will provide a framework that allows us to derive the risk profile of sales from historical sales data.

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