A Practitioners Toolkit on Valuation

Part III: Discount Rates and Financing Policy when Excepted Free Cash Flows have Constant Growth: APV, WACC and CFE

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1 Introduction

In Part I of this toolkit on valuation we showed the consistency of the Adjusted Present Value, the WACC and the Cash Flow to Equity as valuation methods, for a firm with constant (non-growing) expected Free Cash Flows. In particular, we showed that the three valuation methods are consistent with each other, provided that the cost of equity (and therefore the WACC) are (un)levered in a way that is consistent with the firm's financing policy. If the financing policy is to have a constant *level* of debt, then - in the absence of growth - the relation between the unlevered (k_U) and levered (k_E) cost of equity is given by

$$k_E = k_U + \frac{D}{E}\left(1 - T_C\right)\left(k_U - k_D\right). \quad (1)$$

Here E is the value of equity, D is the value of debt, T_C is the corporate tax rate and k_D is the cost of debt. If, on the other hand, the financing policy is to have a constant Debt/Equity *ratio* then the relationship is given by

$$k_E = k_U + \frac{D}{E}\left(k_U - k_D\right). \quad (2)$$

Armed with these two relations, our task in this second part is to analyze the firm with a (constantly) growing Free Cash Flow. We will show that in case of a constant debt policy, the WACC and the cost of equity need to be adjusted, to reflect the fact that a constant level of debt implies that the Debt/Equity ratio is expected to change due to growth. In addition, the growth rate of the Cash Flow to Equity needs to be adjusted as the growth in the Free Cash Flow is levered in the Cash Flow to Equity. To be precise, the cost of equity in (1) and the growth in the Cash Flow to Equity need to be adjusted as follows:

$$k_E = k_U + \frac{D}{E}\left(1 - T_C\right)\left(k_U - k_D\right) + g_0 \frac{D}{E} T_C, \quad (3)$$

$$g_E = g_0\left(1 + \frac{D}{E}\right), \quad (4)$$

where g_0 is the growth in the Free Cash Flow and g_E is the growth in the Cash Flow to Equity.

When the financing policy is to have a constant Debt/Equity ratio on the other hand, we will show, that there is no need to adjust the discount rates and growth rates, and the same discount rates can be applied as in the no-growth case (i.e, Equation (2) still applies).

Together with Part I of this article the purpose is to provide the reader with a framework on how the APV, WACC and CFE-methodologies can be used in a consistent way. Key to applying these frameworks is to understand the financing policy of the firm: is this primarily based on a constant level of the debt, or is it based on a constant Debt/Equity ratio? Depending on this policy choice, the cost of equity (and thus the WACC) as well as the growth rates used in the APV, WACC and CFE methodologies may need to be (un)levered in different ways - but given these adjustments our claim is that the different valuation methodologies are consistent with each other and (in principle) lead to the same valuation.

Below we will first address the adjustment of the discount and growth rates in the different valuation methodologies when the firm has a constant level of debt. In Section 3 we will repeat that analysis for the cases of a constant Debt/Equity ratio policy. Throughout we will illustrate the calculations with the same example as in Part I of this article. We will conclude in Section 4.

2 The case of constant debt

The effect of financial leverage on growth rates is illustrated with the same example as in Part I, the input parameters for which are repeated in Exhibit 1. The level of debt at the outset is D which in this section is assumed not to change, according to the firm's financing policy. Exhibit 1 also shows the growth for the unlevered Free Cash Flow (FCF), which is $g_0 = 2.0\%$.

Exhibit 1: Input data			
Rf	4.0%	FCF	200.00
Rm-Rf	5.0%	kd*D	50.00
Bu	0.80	kD*D*Tc	15.00
ku	8.0%	Change D	-
kD	5.0%	CFE	165.00
Tc	30%		
g0	2.0%		
D	1000		

2.1 Adjusted Present Value

With a constantly growing FCF, the all-equity or unlevered value of the firm is calculated as

$$V_U = \frac{FCF}{k_U - g_0}. \tag{5}$$

In the numerical example, unlevered beta is 0.80, the risk free rate is 4.0% and the market risk premium is 5.0%, giving an unlevered cost of capital $k_U = 8.0\%$. With an expected FCF in the first year of 200, the unlevered firm value equals $V_U = 200/(8\% - 2\%) = 3,333$.

This unlevered value would equal the value of equity, if there would be no debt. If there is debt however, as is the case in our example, this yields tax savings on the interest payments, the value of which can be added to the unlevered value to obtain total firm value. Thus, the Adjusted Present Value of the firm is:

$$V = V_U + TS, \tag{6}$$

where TS is the value of the tax shield. As explained in Part I, in case of constant debt the value of the tax shield is[1]

$$TS = D \times T_C, \tag{7}$$

which in our example equals $1,000 \times 30\% = 300$. Total company value based on the Adjusted Present Value methodology is therefore 3,633, which consists of 1,000 debt and 2,633 equity. This is summarized in Exhibit 2.

We now proceed to show how the same values for the firm and its equity can be obtained by adjusting the discount rates and growth rates applied in the WACC and CFE-methodologies.

Exhibit 2: Firm Value using the Adjusted Present Value Constant Debt			
Vu	3,333.33	E	2,633.33
TS	300.00	D	1,000.00
V	3,633.33	V	3,633.33

2.2 Weighted Average Cost of Capital

The Weighted Average Cost of Capital (WACC) is the discount rate that, when applied to the Free Cash

[1] We assume the level debt to be constant - the approach here can be generalized further by also allowing for a fixed growth in the level of debt, in which case the tax shield will also grow.

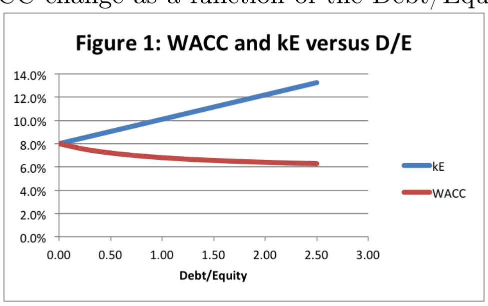
Flow, gives total firm value. The WACC is commonly calculated as

$$WACC = \frac{E}{V}k_E + \frac{D}{V}(1 - T_C)\, k_D. \quad (8)$$

To value the total company, this WACC can be applied to the Free Cash Flow with the same growth rate of the Free Cash Flow g_0:

$$V = V_U + TS = \frac{FCF}{WACC - g_0}. \quad (9)$$

With a constant level of debt and growth in the Free Cash Flow, it is important to note that in Equation (8) we can no longer expect that the Debt/Equity ratio (and thus E/V and D/V) remains constant. This is a fundamental difference with Part I of this article, where in the no-growth case we did expect the Debt/Equity ratio to remain unchanged[2]. Figure 1 shows how (based on Equations (1) and (8)) the cost of equity and WACC change as a function of the Debt/Equity ratio.



From this figure it is clear that as the Debt/Equity ratio falls, the cost of equity will decrease and the WACC will increase. Thus, when applying the WACC, Free Cash Flows that are farther in the future should be discounted at a higher rate than Free Cash Flows in the near future. However, our task here is to find a single WACC that can be applied to all expected Free Cash Flows.

In order for the value in (9) to indeed be consistent with the Adjusted Present Value in (4), the cost of equity underlying the WACC should be calculated as in (3)[3]:

$$k_E = k_U + \frac{D}{E}(1 - T_C)(k_U - k_D) + g_0 \frac{D}{E} T_C.$$

Substituting this into (8), we find for the WACC itself:

$$WACC = k_U - (k_U - g_0)\frac{D}{V} T_C. \quad (10)$$

Thus, Equation (10) shows that in order to apply the WACC as a discount rate to a growing Free Cash Flow, we need to adjust the WACC upwards by the ratio of the tax shield to total firm value times the growth rate. This upward adjustment reflects the fact that the WACC is actually increasing as the Debt/Equity ratio is expected to decrease over time. We can think of the WACC in (10) as a disount rate applied to all future cash flows in the same way as the yield to maturity on a bond: although coupon payments and the face value of a bond should all be discounted at the interest rates associated with the different maturities of the coupons and face value (i.e., according to the term structure of interest rates), discounting them at the single yield to maturity gives the same bond value. In the same way, although the Free Cash Flows for different periods should be discounted at different WACCs (i.e., according to a term structure of WACCs), we can equivalently apply the single WACC in (10) to all cash flows. This single WACC is then essentially a time-weighted average of the term structure of WACCs. It is important to emphasize here that the WACC itself is not a real cost of capital either - it certainly isn't the weighted cost of capital required by equity and debt-holders, as both of them are interested in the total return, not the return after taxes paid by the firm. Rather, the WACC is merely a discount rate that - when applied to the Free Cash Flow - gives total firm value (accounting for the capital structure).

In our example, the WACC in (10) is:

$$WACC = 8.0\% - (8\% - 2\%) \times \frac{1000}{3,633} \times 30\% = 8.5\%.$$

[2] To be precise, we did not assume that the Debt/Equity ratio is constant, but with no growth, our expectation of the Debt/Equity ratio is constant.

[3] The derivations of these results are given in the Appendix.

Applied to the Free Cash Flow of 200, this yields total firm value of 200/(8.5% - 2.0%) = 3,633, the same as in the Adjusted Present Value calculation in Exhibit 2.

2.3 Cash Flow to Equity

The equity of the company can also be valued directly by discounting the Cash Flow to Equity (CFE) at the cost of equity, adjusted for the appropriate growth rate g_E. The CFE is the Free Cash Flow, adjusted for the cash flow from debt (including tax savings):

$$CFE = FCF - k_D\,(1 - T_C)\,D + \Delta D. \tag{11}$$

Since the level of debt is constant in the current setting, $\Delta D = 0$. In case the level of debt is constant, the cost of equity is given by (3). When the FCF is growing at a rate g_0, it can be shown that the growth rate applied to the CFE is[4]:

$$g_E = g_0 \left(1 + \frac{D}{E}\right). \tag{12}$$

Equation (13) levers g_0 according to the Debt/Equity ratio. This makes sense, in that the CFE will be growing at a faster rate than the FCF itself when it is levered by debt that itself is not growing. The growth rate in the CFE is than multiplied by a leverage factor that reflects the financial structure of the company.

In our example the Cash Flow to Equity and the corresponding growth rate are

$$\begin{aligned} CFE &= 200 - 5\% \times 70\% \times 1000 = 165, \\ g_E &= 2\% \left(1 + \frac{1000}{2,633}\right) = 2.8\%. \end{aligned}$$

The cost of equity according to (3) is

$$\begin{aligned} k_E &= 8\% + \frac{1000}{2,633} \times 70\% \times (8\% - 5\%) \\ &\quad +2\% \times 30\% \times \frac{1000}{2,6333} = 9.0\%, \end{aligned}$$

so the value of equity is 165/(9.0% - 2.8%) =2,633, as it is in Exhibit 2.

[4] This follows from $V = D + E$. When the firm grows at a rate g_0 and debt is constant, we get $(1 + g_0)\,V = D + (1 + g_E)\,E$. Solving for g_E gives (13).

3 The case of a constant Debt/Equity ratio

Proceeding with the same firm, that has a constant growth g_0 of the Free Cash Flow, we now analyze the case in which the financing policy is to have a constant Debt/Equity ratio rather than a constant level of debt. As explained in Part I of this article, when the firm has a constant Debt/Equity ratio, the approriate discount rate that needs to be applied to the tax savings is not the cost of debt k_D, but the unlevered cost of capital k_U. As both debt and equity are continuously adjusted when the unlevered value V_U changes, so will the value of the tax shield TS. The same logic applies to the growth rate. If the unlevered value of the firm, or the Free Cash Flow, grows at a rate g_0, then both the debt and equity must grow at the same rate in order to keep the Debt/Equity ratio constant, and thus the same growth rate must also apply to the tax shield.

3.1 Adjusted Present Value

When both the Free Cash Flow and the tax savings are growing at the same rate g_0, the Adjusted Present Value is determined as follows:

$$V = V_U + TS, \tag{13}$$

$$V_U = \frac{FCF}{k_U - g_0}, \tag{14}$$

$$TS = \frac{k_D \times D \times T_C}{k_U - g_0}. \tag{15}$$

In our example this means that

$$\begin{aligned} V_U &= \frac{200}{8.0\% - 2.0\%} = 3,333 \\ TS &= \frac{5\% \times 1000 \times 30\%}{8.0\% - 2.0\%} = 250. \end{aligned}$$

Total company value using the APV therefore equals 3,333+250=3,583, consisting of 1,000 debt and 2,582 equity. This is summarized in Exhibit 3.

Exhibit 3: Firm Value using the Adjusted Present Value			
Constant Debt/Equity ratio			
Vu	3,333.33	E	2,583.33
TS	250.00	D	1,000.00
V	3,583.33	V	3,583.33

3.2 Weighted Average Cost of Capital

When using the Weighted Average Cost of Capital, the Free Cash Flows are disounted at

$$WACC \quad = \quad k_U - \frac{D \times T_C}{V} k_D. \qquad (16)$$

Since all items on the balance sheet are growing at the same rate, this is the same result as in the no-growth case discussed in Part I of this article.

In our example, we now get for the WACC

$$WACC = 8.0\% + \frac{1,000 \times 30\%}{3,583} \times 5\% = 7.6\%.$$

Applying this WACC to the Free Cash Fow of 200 together with a growth rate of 2%, results in 200/(7.6%-2.0%) = 3,583 as in the Adjusted Present Value calculation in Exhibit 3.

3.3 Cash Flow to Equity

As a last valuation method we again discuss the Cash Flow to Equity. With a constant Debt/Equity financing policy, debt is also expected to grow at a rate g_0, so $\Delta D = g_0 D$, which increases the CFE in (12). This CFE should now be discounted at

$$k_E = k_U + \frac{D}{E}(k_U - k_D),$$

and the growth rate for CFE is also $g_E = g_0$, as the changing debt level implies there is no additional leverage in the growth rate.

In our example, this gives

$$\begin{aligned} CFE &= 200 - 5\% \times 70\% \times 1000 + 2\% \times 1000 = 185, \\ k_E &= 8.0\% + \frac{1,000}{2,583}(8.0\% - 5.0\%) = 9.2\%, \end{aligned}$$

implying that total equity is 185/(9.2%-2.0%)=2,583, as is also shown in Exhibit 3.

4 Conclusions

In Part I of this article we showed that the APV, the WACC and the CFE methodologies for valuing companies with no expected growth in the Free Cash Flow lead to the same result, provided that the cost of equity and the tax shield are adjusted in the right way. In this part we have shown how to adjust the discount rates (equity, k_E, and $WACC$) as well as the growth rate in the Cash Flow to Equity, so they can also be used in case the Free Cash Flow is expected to grow at a constant rate, g_0.

In particular, if the financing policy is such that the level of debt is constant, the discount rates to be used when the FCF is discounted at the WACC needs to be adjusted with an adjustment factor that relates the WACC and k_E to k_U with the growth rate. Likewise, the growth rate applied to the CFE is levered with the Debt/Equity ratio. This reflects the fact that as debt does not grow, whereas FCF (and thus unlevered value) does grow, equity grows faster because of financial leverage.

When the financing policy is such that the Debt/Equity ratio is constant, no adjustment to the discount rates and growth rates is needed, as unlevered value, tax shield, debt and equity are all expected to grow at the same rate. In this case the results are the same as without growth.

References

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