A Practitioners Toolkit on Valuation

Part II: (Un)Levering Betas

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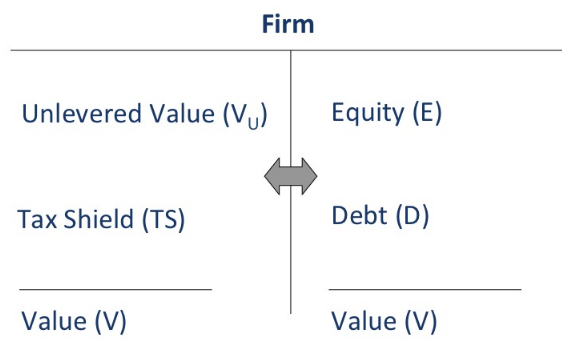
1 Introduction

When valuing companies using the Discounted Cash Flow framework, we usually use the Capital Asset Pricing Model (CAPM) or a related model to determine the appropriate discount rate for Equity, Debt, and the Assets. For any security, the CAPM implies that the appropriate discount rate (or required or expected return) k is determined by

$$k = Rf + \beta (Rm - Rf), \quad (1)$$

where Rf is the risk free (interest) rate, Rm is the expected return on the market portfolio (and thus $Rm - Rf$ is the market risk premium), and β is the security's beta with respect to te market.[1] In valuation exercises we often need to derive the Unlevered or Asset beta from the betas of Equity and Debt of publicly traded (comparable) firms, or we may need to find the Equity beta from the Asset beta taking into account the appropriate leverage of the company. The purpose of this article is to show the relationship between the Asset or Unlevered beta on the one hand, and the betas of Equity and Debt (the liabilities) on the other hand.

To be precise, we have the following market-value based balance sheet of a company in mind:



Each element on this balance sheet has its own discount rate or required return k. On the right hand side, Equity requires a return k_E and Debt requires a return k_D, whereas on the left hand side, the unlevered firm requires a return k_U and the Tax Shield a return k_{TS}. From the CAPM in (1), it follows that each element has its own β (β_E,β_D,β_U, and β_{TS} respectively). Of course, as this is a balance sheet, the sum of the right hand side and left hand side need to be equal, and this also holds for income streams and systematic risk, or beta:

$$V_U \times k_U + TS \times k_{TS} = E \times k_E + D \times k_D, \quad (2)$$
$$V_U \times \beta_U + TS \times \beta_{TS} = E \times \beta_E + D \times \beta_D. \quad (3)$$

When doing the valuation, the starting point is always the Free Cash Flow that is generated by the firm. From the balance sheet above, there are at least two ways to do the valuation: the Adjusted Present Value (APV) method or the Weighted Average Cost of Capital (WACC) method[2]. The Adjusted Present

[1] Formally, the beta of security i is defined as $\beta_i = Cov[R_i, Rm]/Var[Rm]$.

[2] In Part I of this series we also discussed the Cash Flow to

Value method essentially values the two parts of the left hand side, V_U and TS, separately, and adds them to obtain total value V. The Unlevered Firm value, i.e., the value that we would obtain if there would be no debt on the balance sheet whatsoever, is obtained by discounting the Free Cash Flows at the unlevered cost-of-capital:

$$V_U = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+k_U)^t}. \tag{4}$$

Adding the value of the Tax Shield to this unlevered value yields total firm value. The WACC method starts from the same Free Cash Flows, but discounts them at a rate that immediately yields total firm value:

$$V = \sum_{t=1}^{\infty} \frac{FCF_t}{(1+WACC)^t}, \tag{5}$$

$$WACC = \frac{E}{V} \times k_E + \frac{D}{V} \times (1-T_C) \times k_D. \tag{6}$$

In the WACC we adjust the cost-of-debt k_D for the corporate tax rate, T_C, to incorporate the Tax Shield in the valuation directly.

In part I of this Toolkit series we showed that the APV and WACC methods are entirely consistent, but that the exact implementation (i.e., the exact formulas) to be used depend on the financial policy of the firm. Specifically, the exact cost-of-capital (WACC and k_E) to be used depend on whether the firm has i) a fixed Debt policy (i.e., the *level* of Debt is always known) or ii) a fixed Debt-to-Equity policy (i.e., the *ratio* Debt/Equity is always known). In the sequel of this article we make the same distinction in financial policy to relate the different betas to each other.

2 The case of constant Debt

As discussed in Part I of this series, when the level of Debt is constant, or known, the tax saving that arises from the use of Debt is also known, and the risk of (not) realizing the tax savings is equal to the risk of (not) realizing the Debt payments. Thus, the appropriate discount rate for the Tax Shield is the same as for Debt, k_D. This implies (Equation (8) of Part I) that the WACC can be written as

$$WACC = \left(1 - \frac{D \times T_C}{V}\right) k_U.$$

Using the expression of the WACC in (6) and multiplying both sides of the equation with total firm value V gives

$$E \times k_E + D \times (1-T_C) \times k_D = (V - D \times T_C) \times k_U,$$

which can be solved for k_U as:

$$k_U = \frac{E}{V_U} \times k_E + \frac{D \times (1-T_C)}{V_U} \times k_D, \tag{7}$$

where it should be noted that $V_U = E + D \times (1-T_C)$. Thus, Equation (7) says that the unlevered cost-of-capital is a simple weighted avarage of the cost-of-equity, k_E, and the cost-of-debt, k_D, where the weights are Equity and after-tax Debt as a fraction of the Unlevered Value. Since the weights in (7) sum to one, it easily follows that the same applies for the betas: the unlevered beta, or asset beta, equals:

$$\beta_U = \beta_A = \frac{E}{V_U} \times \beta_E + \frac{D \times (1-T_C)}{V_U} \beta_D. \tag{8}$$

3 The case of a constant Debt/Equity ratio

When the ratio of Debt to Equity, D/E, is constant, the firm needs to adjust its level of Debt all the time as its basic value V_U changes. This means that the tax savings resulting from Debt are changing with V_U as well, and that the riskiness of the tax savings is the same as the risk in V_U. The appropriate discount rate associated with the Tax Shield is therefore k_U. This in turn implies (Equation (11) of Part I) that the WACC can be written as

$$WACC = k_U - \frac{D \times T_C}{V} \times k_D.$$

Proceeding again with the WACC in (6) and multiplying both sides of the equation with total firm value V now gives

$$E \times k_E + D \times (1-T_C) \times k_D = V \times k_U - D \times T_C \times k_D.$$

Equity method as a third alternative.

This is easily solved for k_U as

$$k_U = \frac{E}{V} \times k_E + \frac{D}{V} \times k_D. \quad (9)$$

As in (7), the unleverd cost-of-capital is again a weighted average of the cost-of-equity, k_E, and the cost-of-debt, k_D, but now we use the total Debt instead of the after-tax level of Debt, and we express the weights as a fraction of total firm value rather than its unlevered value. These weights also sum to one, and therefore we also have for the unleverd beta, or asset beta:

$$\beta_U = \beta_A = \frac{E}{V} \times \beta_E + \frac{D}{V} \times \beta_D. \quad (10)$$

In practice, in valuing a company, we often start from the Equity betas, β_E, of companies that are similar to the company we are analyzing, for instance by looking at listed companies from the same industry. An important step is then to unlever the observed (or estimated) Equity beta of these comparable firms, which can be done by either Equation (8) or (10). The valuator therefore has to ascertain himself which financing policy is the relevant one for the comparable firms: a fixed Debt policy or a fixed Debt/Equity policy. For many mature (listed) firms, the relevant case will often be the latter. Next, when relevering the Equity beta from the Asset beta for the firm valuation at hand, the Valuator has to determine the financial policy of the firm to be valued - and then apply (8) or (10) depending on the policy.

4 The cost-of-debt and β_D

So far we explicitly allowed for Debt to have a nonzero β_D, and thus for the cost-of-debt to exceed the risk free interest rate, i.e., $k_D > Rf$. The difference between the cost-of-debt and the risk free interest rate is mostly related to the credit spread (i.e., a compensation for default risk), which is normalle captured by models for credit risk. For the purpose of this article, it is most convenient to capture the credit risk premium in the context of the CAPM though, i.e., by having a nonzero β_D. In many text books this β_D is assumed to be zero, in which case the relationship between the Unlevered and Equity betas in (8) and (10) respectively, obviously simplify to the well known formulas

$$\beta_E = \frac{V_U}{E}\beta_U = \left(1 + \frac{D \times T_C}{E}\right)\beta_U, \quad (11)$$

$$\beta_E = \frac{V}{E}\beta_U = \left(1 + \frac{D}{E}\right)\beta_U. \quad (12)$$

Although these formulas are widely used, it is important to stress that these assume (explicitly) that there is no risk premium in the cost-of-debt, i.e, that $k_D = Rf$.

5 Extension to alternative asset pricing models

The analysis above started from the simple CAPM, where there is only one beta for each security or asset. There are many alternative asset pricing models to the CAPM, which often have multiple betas. These models are also known as multi-factor models. One famous example of this is the Fama-French model (Fama & French (1996)), which says that expected (or required) stock returns are not described by the CAPM in (1), but by a three-factor model:

$$k = Rf + \beta \times (Rm - Rf) + s \times SMB + h \times HML. \quad (13)$$

Here, SMB is the expected return on stocks of Small companies versus Big companies (Small-Minus-Big), measured by the size of the market value of their equity, and HML is the expected return on companies with a High Book-to-Market ratio (of equity) versus companies with a Low Book-to Market ratio (High-Minus-Low). Like β is the exposure to market risk, s and h are the exposures to Size and Book-to-Market risk respectively. The intuition of the Fama-French model is based on the idea (and empirical findings) that Small firms demand a higher return than Big firms, for instance because they are less liquid, have less analyst coverage, or are more similar to non-traded (private) firms. This is reflected in the Small-firm premium SMB. Firms that behave like Small firms have a high SMB-exposure s and therefore command a higher risk premium $s \times SMB$ similar to $\beta \times (Rm - Rf)$. Likewise firms with a High

Book-to-Market ratio demand a higher return than firms with a Low Book-to-Market ratio, for instance because these low-priced stocks indicate higher credit risk. This higher credit risk is reflected in the Book-to-Market premium HML. This induces a risk premium $h \times HML$ for firms, depending on their exposure to Book-to-Market risk.

When applying the Fama-French model to determine the discount rate in a valuation, the relation between the Unlevered (or Asset) exposores s and h to the Equity and Debt exposures is equivalent to Equations (8) and (10). For the case of constant Debt we have

$$\begin{aligned} s_U &= s_A = \frac{E}{V_U} \times s_E + \frac{D \times (1 - T_C)}{V_U} s_D, \\ h_U &= h_A = \frac{E}{V_U} \times h_E + \frac{D \times (1 - T_C)}{V_U} h_D. \end{aligned}$$

For the case of a constant Debt/Equity ratio, we likewise have

$$\begin{aligned} s_U &= s_A = \frac{E}{V} \times s_E + \frac{D}{V} s_D, \\ h_U &= h_A = \frac{E}{V} \times h_E + \frac{D}{V} h_D. \end{aligned}$$

The important thing is that with these three factors, the relation between the exposures on the Assets, Equity, and Debt is equivalent to the case of the CAPM. This result applies to all asset pricing models that are expressed as (multiple) beta models.

6 Conclusions

In this second part of our practitioner's toolkit on valuation we addressed the issue how to (un)lever betas. The important insight is that the valuator first needs to identify the financial policies of both the comparable firms that are used to estimate the Asset or Unlevered beta, and also needs to determine the financial policy of the company that is being valued, to get the levered Equity beta. Depending on whether the financial policy is to have a fixed (or known) *level* of Debt or a fixed Debt/Equity *ratio*, betas need to be (un)levered in different ways. This is similar to the consistency between different valuation methods as discussed in Part I of this toolkit. Although we have illustrated the (un)levering of betas within the framework of the CAPM, the (un)levering works in the same way for other (multiple) beta pricing models.

7 References

Fama, E.F., and French, K.R., 1996, "Multifactor Explanations of Asset Pricing Anomalies", *The Journal of Finance*, *LI* (1), p.55-84.