Why *listed equity* real estate?

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Empirical insights into its role within
a mixed-asset portfolio

Master Thesis

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Abstract

The global listed equity real estate markets are growing and are becoming a more important part of the financial markets. Nevertheless, many investors and academics cannot properly sort an investment within a mixed-asset portfolio yet. Therefore, this thesis examines the characteristics of listed equity real estate to evaluate its contribution as a distinct asset class to a mixed-asset portfolio of an US investor. The study discusses the return enhancement, diversification, and inflation hedging capabilities of passive investment strategies on a national and global level. The empirical tests led to the following conclusions. First, risk and performance metrics indicate high risk compared to other asset classes. Nevertheless, absolute return enhancement capabilities for less risk-averse investors are identified. Second, the detailed correlation analysis reveals promising global diversification possibilities but increasing correlations during times of turmoil. Third, spanning and out-of-sample tests are contradicting and weaken the idea of economical and statistical advantages of adding listed equity real estate. Fourth, indications for long-term inflation hedging and time diversification capabilities are found. Next to these findings, the study contributes to both financial research and practice and identifies avenues for future research.

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List of Abbreviations

ADF	Augmented Dickey–Fuller
BLS	Bureau of Labor Statistics
CAPM	Capital Asset Pricing Model
CMBS	Commercial mortgage-backed securities
CPI	Consumer Price Index
DCC	Dynamic Conditional Correlation
ECT	Error Correction Term
EPRA	European Public Real Estate Association
ES	Expected Shortfall
EWMA	Exponential weighted moving Average
GICS	Global Industry Classification Standard
GPR	Global Property Research
HML	High minus Low
i.i.d	independent and identically distributed
MCAP	Market Capitalization
MOM	Momentum
NAREIT	National Association of Real Estate Investment Trusts
PP	Phillips-Perron
REIT	Real Estate Investment Trust
REOC	Real Estate Operating Company
SD	Standard Deviation
SMA	Simple Moving Average
SMB	Small minus Big
GSCI	Goldman Sachs Commodity Index
VaR	Value at Risk

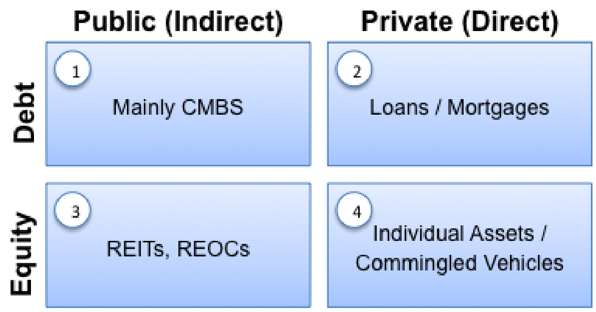
1 Introduction

In 1960, Real Estate Investment Trusts (REITs) were created in the US with the intention to give retail investors the opportunity to easily invest in diversified real estate portfolios. In the beginning, the industry was growing rather slow, which is why further regulative changes were needed to boost the sector. The REIT Modernization Act finally permitted REITs to manage their own properties. This act in combination with the Revenue Reconciliation Act in 1993 paved the way for explosive growth of commercial listed equity real estate (Dirk Brounen & de Koning, 2012). Since then, REIT regimes with similar characteristics have been created in over 30 countries and the market has been experienced a tremendous growth.[1] The National Association of Real Estate Investment Trusts (NAREIT) reports that in the US alone REITs currently own properties worth over $1 trillion and that listed equity REITs have a combined market capitalization (MCAP) of over $846 billion. Idzorek *et al.* (2006) even identify an ongoing shift from private commercial real estate to listed commercial real estate because of the higher liquidity and transparency in the market environment.
Due to the growing interest in the market many national and global indices have been created following the inception of the NAREIT REIT index in 1972.[2] The creation of the indices enabled the development of many index-based products. This trend should be further boosted by the recent announcement of S&P Dow Jones Indices and MSCI Inc. to include real estate as a 11th headline sector of the global industry classification standard (GICS) in August 2016. The newly developed index-based products in combination with the growing number of specialized mutual funds enable investors to further diversify their portfolio with a broad passive investment in real estate. Thus, the idea of an additional diversified real estate portfolio becomes more appealing and feasible.[3] However, many retail and also institutional investors are not sure about the role of an explicit investment within their portfolios yet.

The scientific community also reacted to the growing importance of public real estate in the financial markets. In 2003, the Journal of Portfolio Management set a milestone and published its first special Real Estate Issue, which was entirely devoted to the analysis of real estate investments. In this issue Hudson-Wilson *et al.* (2003) identify potential benefits of adding real estate to a mixed-asset portfolio in their article “Why Real Estate?”. In 2005, a revised version has been published but still leaves many questions unanswered and room for critique. First, the empirical work is based on 17 years of quarterly data, which equals only 68 observations. Furthermore, the occurrence of the financial crisis might have changed the perception of an investment. Therefore, their results are not statistically robust and not up to date. Second, the formed cap-weighted real estate index does not properly reflect the true investable universe and mingles different forms of a real estate investment (see Fig. 1).

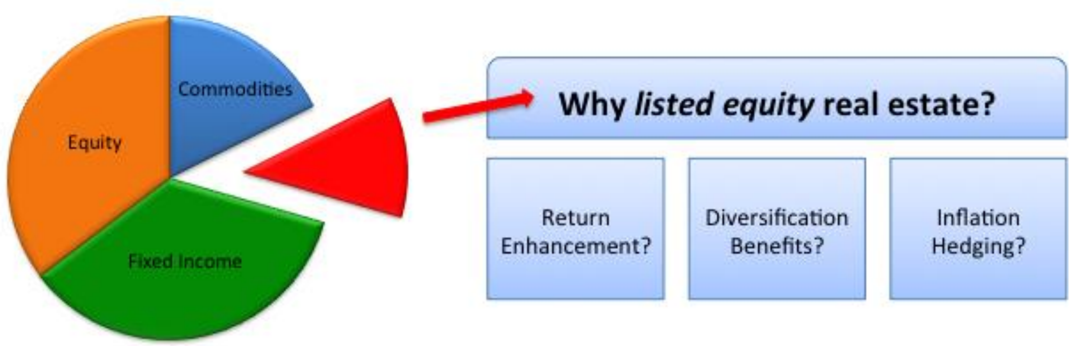
[1] For a summary of different REIT Regimes around the world see Exhibit 1 in Brounen and de Koning (2012).
[2] Based on the market capitalization of the GPR General Quoted Index the global market has grown to a market capitalization of over $1.6 trillion in the beginning of 2015.
[3] NAREIT provides a list of global funds: https://www.reit.com/investing/investing-tools/global-funds.

Fig. 1:
Commercial Real Estate Quadrants



From an asset management perspective it is more desirable to focus on one of the four quadrants in order to isolate the analysis and to get more reliable and useful results. Finally, their study is limited to investments in the United States and does not consider the possibility of international investments. Despite the fact that several studies addressed parts of the issues and only focused on the first quadrant of Fig. 1, they only examined specific characteristics of listed equity real estate. An updated overview similar to the study of Hudson-Wilson *et al.* (2005), focusing only on listed equity real estate, is more desirable. Thus, the aim of the research is to address these limitations in order to equip academics and practitioners with a comprehensive examination of a listed equity real estate investment. For that, an overreaching research question is formulated: Can listed equity real estate contribute to an already well-diversified mixed-asset portfolio? In order to answer this research question from the point of view of a national, as well as global, portfolio manager, three subquestions can be formulated (see Fig. 2).

Fig. 2:
Research Question and Subquestions



First, can an investment act as a return enhancer with high mean returns or superior risk-adjusted returns? Second, might an investment help to further diversify the existing portfolio through low correlations with other asset classes? Third, is a better protection against inflation possible?[4]

[4] Hudson-Wilson *et al.* (2005) further consider the ability of delivering strong cash flows and the importance as part of a market neutral portfolio. The former point should be redundant, as the study considers total return indices.

In order to answer the formulated subquestions, the analysis is divided in two main parts: a pure national and a global analysis from the perspective of a passive US investor. In both parts, each of the three potential benefits are discussed individually and finally brought together in a connecting portfolio optimization framework. Moreover, the international analysis is conducted in two different ways. First, broad available indices relying on their market weights are used. Second, a selection of country indices is employed, which allows investors to deviate from the country weights of the broader indices. This distinction is necessary as in many countries the market for listed real estate is still in a growing phase and therefore weights based on the MCAP might not properly reflect an optimal allocation. Furthermore, additional analyses of subsamples are conducted, as several studies indicate a change in potential benefits and a further integration of the global property markets.[5] However, the point of division of the sample is still debatle and the static presentation of the paper highly limits the capability of the reader to scrutinize the results. To solve this issue, the paper additionally offers the reader an accessible website to further investigate parts of the empirical tests in a dynamic framework.[6]

The results of the study show that investors should not be too overoptimistic about additional benefits from a distinct investment in listed equity real estate if they already have a well-diversified mixed-asset portfolio. In particular, contributions in form of risk-adjusted returns are questionable, due to the high risk of an investment. Nevertheless, several periods indicate the possibility to use an investment as an absolute return enhancer. Furthermore, a global investment can help to further diversify a portfolio and to get exposure to real estate unique risk factors. Additionally, long-term investors should be able to benefit from time diversification and inflation protecting capabilities of listed equity real estate.

The remaining parts of the paper are structured as follows. The second section reviews the most relevant literature. Here, research gaps are identified and implications for future analysis are derived. In the third section, an overview and evaluation of the used data is given. The fourth section discusses advantages and disadvantages of methodologies in order to identify the best complementing set. Afterwards, the results are presented and discussed in section five. At the end of this section, a short summary of the main results and a comparison to the reviewed literature is given. Finally, the last chapter gives a brief conclusion and makes some remarks about the conducted tests.[7]

[5] The tax reform act in 1993 is often seen as the most important structural change in the REIT market (e.g. Glascock *et al.*, 2000, M.-L. Lee *et al.*, 2008).
[6] The website can be accessed via the following address: https://bernhardschlepper.shinyapps.io/MasterThesis
[7] In the following the term real estate is used as an equivalent of listed equity real estate. If listed equity real estate is not meant, it is made clear for the reader.

2 Literature Review

Many years have passed since the introduction of REITs and with them a lot of research about the role of real estate as an explicit investment in a portfolio has been conducted. For example, Worzala and Sirmans (2003b) give an ample overview of studies.[8] Intriguingly, many studies found that an equity real estate investment can indeed contribute and that the traditional allocation might be too low. As investors are interested in the performance, risk and correlations conditional on all available information, the study gives an overview of more recent studies. This is important as several studies showed that the characteristics changed over the last few years, as there have been many structural changes in real estate markets (Chong *et al.*, 2009). Furthermore, disrupting events like the dotcom bubble, the financial crisis, and the European sovereign debt crisis might have changed the picture and should help to get insights about the reactions to turmoil markets. A reassessment of the risk return characteristics of real estate is needed (Moss & Baum, 2013).

The overview is organized as follows. Firstly, an understanding of the connection between a direct and an indirect investment is necessary, as an investment should get an exposure to the direct real estate market. Therefore, a brief impression about the debate of the cointegration of the two types of investment is presented in the first paragraph. Secondly, recent papers discussing the return and the involved risk of a real estate investment are introduced. Thirdly, studies about diversification possibilities of an investment are discussed. This section is subdivided into three areas, as it distinguishes between different dimensions of diversification. Finally, research about the inflation capabilities of real estate is presented in the fourth paragraph.

Direct vs. indirect Real Estate Investment

Direct real estate investments carry several well-known issues such as low liquidity, the size of the absolute investment, low transparency on a corporate level, high transaction costs, and barriers for international investments.[9] These problems could be cured through listed equity real estate as long as the returns are not just driven by the general stock market and a common "real estate factor" determines both forms of investment. Then, the well-documented diversification benefits for direct investments should also pertain for indirect investments (e.g. Worzala & Sirmans, 2003a).

As the bulk of the income of REITs and Real Estate operating companies (REOCs) has to come from property related activities to qualify for the classification, returns of an investment should highly depend on the underlying real estate market. However, the problem is that in several time periods low contemporaneous correlations between direct and indirect investments can be observed. It rather seems like listed real estate returns contain a significant stock market noise (Hoesli & Oikarinen, 2012). On the other hand, the market for direct real estate investments is quite slow when it comes to adjusting prices to fundamental changes. This is mainly due to the infrequent appraisal based valuation. Bekkers *et al.* (2009) and Idzorek *et al.* (2006) even state

[8] The European Public Real Estate Association (EPRA) provides a frequently published informative report, which summarizes recent research.

[9] Jones Lang LaSalle offers an interactive representation of their global real estate transparency index: http://www.jll.com/GRETI

that the return data of listed real estate indices is superior, as the infrequent valuations lead to smoothing biases and downward biased volatilities. Thus, the direct market might be classified as inefficient, which could be taken as an explanation for relatively low short-term correlations. In this context, many studies argue that the securitized real estate market actually leads the direct real estate market, but both markets adjust to fundamental shocks in the same way in the long run.[10] This has been shown by many studies for different local markets by means of error correction models or cointegration tests (see Giliberto, 1990; Yunus *et al.*, 2012; Oikarinen *et al.*, 2011; Hoesli & Oikarinen, 2012; M.-L. Lee *et al.*, 2008; Sebastian & Zhu, 2012; and Ciochetti *et al.*, 2015).[11] There are only a few older studies, disputing a relationship over different time horizons. However, most of the more recent studies found a connection between the two markets, even after controlling for several macroeconomic variables (Hoesli & Oikarinen, 2012).

In sum, an indirect investment should be a proper substitute for a direct investment especially from a long-term investors perspective (Oikarinen *et al.*, 2011, Hoesli & Oikarinen, 2012). Nevertheless, it should be noted that listed real estate is exposed to market forces. The sector is booming and demand increasing. At the same time supply is rather limited, increasing the likelihood of overvaluations. Furthermore, the returns of real estate companies depend on further characteristics, like leverage and management behavior (Hoesli *et al.*, 2008).[12] These potential weaknesses should be considered when comparing both types of investment and when evaluating the role of listed real estate in a mixed-asset portfolio.

Return Enhancement

Several studies analyzed and compared the risk and performance of listed equity real estate and discussed the capabilities to enhance a portfolio’s return by reducing risk or delivering a higher total return. Benjamin *et al.* (2001) summarize studies, which analyze the performance of REITs but they do not specifically provide information of studies about broad indices. Furthermore, the summarized studies are outdated and are not able to provide a sufficient picture of the performance and risk due to the fluctuations in recent years (S. Lee, 2010). However, by connecting recent studies, using different time frames, some vague conclusions can be drawn.

For the US, several studies conclude that listed equity real estate investments generate higher mean returns than equity but also entail bigger risk, even without considering the burst of the housing bubble in 2008 (Idzorek *et al.*, 2006; D. Brounen & Eichholtz, 2003).[13] Lu *et al.* (2013) point out that the downside risk of less developed REIT markets is on average higher. Nonetheless, they stress that this changed during the crash in 2008 and the most developed markets exhibited higher downside risk. Zhou and Anderson (2012) also conclude that REIT returns generally inhibit higher tail risk than equity returns. This is in line with the cross border tests for alpha of Ling and Naranjo (2002). They cannot find evidence for superior risk-adjusted

[10] In contrast, Tuluca *et al.* (2000) finds the opposite and argues that private real estate investors should be more sophisticated and be able to value the assets more appropriate.
[11] The study of Boudry *et al.* (2012) finds that both private and securitized real estate markets adjust towards each other when using MIT TBI indices, which are not appraisal based. However, most studies find that the private market adjusts towards the leading securitized market.
[12] For instance, the recent paper of Ling *et al.* (2013) finds a connection between returns, their volatility and leverage.
[13] Hoevenaars *et al.* (2008) cannot even identify higher mean returns over their sample.

returns of the GPR indices. Furthermore, it was shown that real estate stocks behave similar to small cap value stocks rather than the stock market as a whole (e.g. Clayton & MacKinnon, 2003). This would imply that the potential return benefits highly depend on the composition of the benchmark portfolio. For instance, S. Lee (2010) points out that since 1999, REITs contribute more through return benefits to a portfolio dominated by large cap stocks than diversification benefits. The contribution to small caps is mixed and REITs cannot provide return benefits to small cap value stocks in both considered time frames. As a passive index investor is assumed and the used indices mainly consist of high cap value stocks, this observation should have a positive impact on the results.

Overall, several studies agree that listed real estate is able to enhance the total return but not risk-adjusted return of a mixed-asset portfolio. However, it is important to highlight that the results highly depend on the analyzed time frame and that the length of the available data is still very limited (Moss & Baum, 2013). An updated comprehensive international analysis is necessary to incorporate recent market turmoil and potential effects on the relative risk-return characteristics of a real estate investment compared to other asset classes.

Risk Diversification

The potential to improve the diversification of a portfolio can be divided in three general dimensions. First, passive investors can try to identify asset classes, which exhibit low correlations. Second, investors can further diversify within a certain asset class by investing across borders (see Kroencke & Schindler, 2012). Third, investors with longer holding periods may benefit from time diversification capabilities. Therefore, the review, similar to Worzala and Sirmans (2003b), is divided into studies related to a mixed-asset portfolios and into studies related to a pure real estate portfolio. Time diversification as a third dimension is discussed in the end of the section.

From the perspective of an US investor, Kroencke and Schindler (2012) conclude that adding international securitized real estate to an existing global equity and bond portfolio leads to a better risk-return trade-off. They further consider the possibility of systematic currency risk and find that the fully hedged mixed-asset portfolio yields an even better risk-return trade-off. Nevertheless, they identify fading diversification benefits of a real estate investment during the recent financial crisis (see also Chong et al. 2012). In contrast, Sa-Aadu *et al.* (2010) emphasize that real estate is useful as a hedge against adverse shocks to consumption growth, particularly in bad times. Chong *et al.* (2009) analyze conditional correlations with a dynamic conditional correlation (DCC) Model and also point out that the correlations between REITs and equity markets for different regional indices increased over their sample period (1990:1-2005:12). This implies decreasing diversification benefits for an equity portfolio manager. The effect seems to strengthen in times of abnormal volatilities when diversification benefits are most desirable (see Lu *et al.*, 2013).[14] In general, Hoesli and Reka (2015) come to the same conclusion. They identify contagion effects between the listed real estate market and the equity market in stressful times. However, they suggest that this observation might be mainly due to behavioral sentiment of

[14] Lu *et al.* (2013) further find the opposite for correlations with bond and commodity indices, which is similar to findings for equity returns.

investors and not necessarily similar risk factors. Nevertheless, investors should be aware of tail dependencies and changing correlation coefficients. Case *et al.* (2012) also highlight the importance of different regimes. They show with a regime-shifting DCC model that the correlation between US REITs and the US equity market changed substantially over the years. Time-varying correlations between the different asset classes are also highlighted by Lizieri (2013).

Similarly, studies for pure real estate portfolio show that the diversification benefits change over time. Liow *et al.* (2015) focus their analysis on the correlations between listed equity real estate markets around the world and emphasize the importance to distinguish between different regimes. In addition to that, they report substantial spillover effects between global real estate markets but don't further investigate the impact on diversification opportunities. This has been done by a study from Lu *et al.* (2013). They come to the conclusion that especially during market bubbles and crashes diversification advantages of an international REIT portfolio drop significantly. Those results in combination with the conclusions of Stevenson (2000) might undermine the previous results of Eichholtz *et al.* (1998) that investors can clearly benefit from diversifying their real estate portfolio internationally. On the other hand very recent studies from Ciochetti *et al.* (2015) and Pavlov *et al.* (2015) point out that real estate returns are clearly driven by local factors, suggesting good global diversification possibilities (see also Ling & Naranjo, 2002).

Finally, investors may also consider the possibility of time diversification, especially if they aim to develop a lifecycle asset allocation strategy or more general a strategy that encompasses a longer time horizon (Bennyhoff, 2008). If asset returns are independently and identically distributed (i.i.d.) over time, the asset allocation will not change with an increasing investment horizon. However, several studies showed that returns for real estate are not i.i.d and advise that a distinction between long-term and short-term allocation is favorable. Time diversification builds on the idea of above average returns offsetting below average returns over long horizons and the belief in mean-reversion (Kritzman, 1994). Hence, a certain degree of predictability in returns can already allow for benefits from time diversification, even without strong indications for mean-reversion (Barberis, 2000). For instance, if the volatility in a real estate investment decreases faster with increasing investment horizon compared to other asset classes, an investor might want to increase his allocation towards real estate (Viceira, 2001).[15] Even though the idea of time diversification sounds compelling, the numbers of studies that cope with time diversification capabilities of listed real estate investments are scarce. This is mainly due to the lack of reliable long time series data (Balvers *et al.*, 2000). There are only two recent studies devoted to the topic. First, Stevenson (2002) uses Variance Ratio tests, Augmented Dickey-Fuller (ADF) tests and portfolio switching tests to examine the momentum effects and potential mean-reversion. His study covers eleven REIT markets around the globe and takes the perspective of a fully hedged US investor over a sample from 1972 to 2004. Overall, he cannot find clear evidence for mean reversion. He also highlights the difficulty to test for mean reversion and pronounces the weaknesses of his tests. His disillusioned findings are substantiated by Fugazza *et*

[15] In particular, young investors may want to increase there allocation as they would be able to compensate short-term loses with their human capital.

al. (2009), who compare optimized portfolios with and without real estate by conducting out-of-sample tests. The optimized portfolios with longer holding horizons seem not to be of any advantage compared to the short-term portfolios.[16]

The mixed results of the reviewed studies highlight the difficulty to evaluate the three dimensions of diversification. Correlations have been changing over time and the lack of sufficient time series data has limited the evaluation of time diversification benefits. Nevertheless, the increasing availability of time series data asks for further studies about the topic.

Inflation Hedging

A final reason for investors to invest in real estate is inflation hedging. However, real estate stocks might only contain hedging characteristics in the long run and a direct investment might be more appropriate as an inflation hedge. That is at least the result of many studies. In addition, most of them deny short-run hedging capabilities and find at most perverse hedging opportunities, as they have been documented for common stock returns (Hoesli *et al.*, 2008).[17]

One example is the study of Liu *et al.* (1997). The authors question a relationship between listed equity real estate and inflation for different countries and can only find perverse hedging capabilities. This observed counterintuitive relationship led to several studies investigating the anomaly. Many of them argue that the perverse hedging capabilities of stock and real estate returns are just due to an omitted variable bias. Fama (1981) was one of the first, who suggested that stock returns and expected inflation alike are influenced by economic variables. Based on money demand theory and the quantity theory of money, he argues, that a positive relation between money demand and anticipated real activity should imply a negative relation between inflation rate and real activity. Thus, the observed negative relation between inflation rate and stock returns seems to proxy for fundamental relations between real activities and stocks. Fama shows with empirical tests that the intuition indeed seems to be true and the negative sign disappears. Such a relationship should also be observable for real estate returns, as they highly depend on future economic activity and are sensible to interest rates. Glascock *et al.* (2002) examine the issue of the perverse hedge of REIT returns and conclude that the issue disappears after controlling for real activities in their tests.[18] This result is confirmed by Hoesli *et al.* (2008) but they cannot find significant coefficients for expected inflation. Nevertheless, their results further indicate that there is a long-term equilibrium relation between returns and expected inflation with a significant and positive sign. Hardin *et al.* (2012) come to the same conclusion and cannot identify short-term hedging capabilities. They explain the perverse hedging characteristics in the short-term with a behavioral approach, called inflation illusion.[19] Following this explanation, investors are not able to adapt their nominal growth rates and nominal discount rates in the short run alike. However, they see this mispricing disappearing in the long run, substantiating the idea of long-term hedging capabilities. This outcome seems to be consistent

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[16] Time diversification benefits of a direct real estate investment might be more likely because of higher liquidity premiums for long-term investors (Rehring, 2012).

[17] Hoesli *et al.* (2008) give good overviews of older studies about the relation of equity and inflation in Table 1 and real estate and inflation in Table 2.

[18] Glascock *et al.* (2002) base their ideas on their previous article Darrat and Glascock (1989).

[19] The theory was developed by Franco Modigliani and Cohn (1979).

across countries. A recent study from C. L. Lee and Lee (2014) compares the inflation hedging abilities of national real estate for different countries. They conclude that for none of the countries real estate can hedge against inflation in the short-run. However, they suggest that there are hedging possibilities against expected inflation for developed countries in the long run. Maurer and Sebastian (2002) conduct a similar study for European countries. They also point out that real estate stocks only seem to be good inflation hedges with an increasing investment horizon but they relate this to high average real returns. A counterexample is a study conducted by Chatrath and Liang (1998). They find only weak evidence for a long-term relationship between a constructed REIT index and Consumer Price Index (CPI).

In summary, most studies neglect short-term capabilities but indicate a long-term equilibrium relationship. Nevertheless, they do not explicitly investigate the role within a mixed-asset portfolio. A direct comparison with equity, bonds, and commodities is missing.

The literature review revealed that the characteristics of real estate changed over time and are sensible to the analyzed time frame. Furthermore, the reviewed studies do not provide a connecting and comprehensive overview of the role of real estate within a mixed-asset portfolio. Therefore, the remainder of this study attempts to overcome the highlighted shortcomings and to give a thorough overview of the role of global real estate within mixed-asset portfolios. This can only be done with the right set of methodologies, which is developed in the following chapter.

3 Methodologies

In order to answer the identified subquestions, this chapter introduces the tools for the empirical analysis. Thus, for each of the four possible benefits tests are developed. First, performance and risk metrics are introduced to compare asset classes and formed portfolios in isolation. Second, methods to calculate correlation coefficients based on covariance matrices are discussed in order to evaluate the diversification potentials of the different asset classes. Third, the portfolio optimization method and spanning tests to assess the statistical significance of the formed portfolios are introduced. Finally, different regression-based approaches to size the inflation hedging capabilities of real estate are discussed.

3.1 Performance & Risk Metrics

In order to compare the risk and return characteristics of real estate indices with other asset classes the study compares descriptive statistics and distribution curves. In addition to that, risk-adjusted returns in form of Sharpe ratios and M Squared are compared. Furthermore, the study estimates value at risk (VaR) and expected shortfall (ES) of the individual indices to get an economical impression of their tail risk. The combination of the complementing risk and performance measures gives a comprehensive overview of the risk and return characteristics of real estate.[20]

One of the most popular performance measures is the Sharpe ratio (W. F. Sharpe, 1994). The ratio divides the excess expected return by the risk measured as the standard deviation (SD) of an investment:

$$SR = \frac{E(r_i) - r_f}{\sigma_i} \tag{1}$$

Thus, the ratio states how much average excess return per unit of risk an asset or portfolio can generate. One drawback of this measure is that users have to specify a risk-free rate, which is a difficult task, particularly in the current low-interest environment. Therefore, the study refers to the return-risk ratio, dividing the whole return by the SD, rather than to the Sharpe ratio. A second problem of the measure is the lack of interpretability.

F. Modigliani and Modigliani (1997) countered this problem by introducing a new risk measure, called M Squared. The measure allows investors to compare the performance in terms of basis points by leveraging or deleveraging the new portfolio until it has the same risk as the benchmark portfolio. This idea can be expressed through the following formula (S. Lee, 2010):

[20] For some of the calculations the functions of the R package "PerformanceAnalytics" from Peterson *et al.* (2014) are used.

$$M^2 = \left[\frac{\sigma_B}{\sigma_N}\left(r_N - r_f\right) + r_f\right] - r_B \qquad (2)$$

The expected excess return of the new portfolio r_N gets scaled by the ratio σ_B/σ_N of the SDs of the two portfolios. After adding back the risk free rate, the return of the benchmark portfolio r_B is subtracted. This leads to the difference of the two compared investments in form of basis points at the same level of risk.

So far the metrics do not attempt to explain where the variation in real estate returns comes from and if there might be parts of the return, which cannot be explained by common risk factors. Jensen (1968) developed a method to measure the performance of mutual funds. His approach is based on the ideas of the capital asset pricing model (CAPM) (Lintner, 1965,William F. Sharpe, 1964 and Treynor, 1961), which states that the expected return of every asset should solely depend on the exposure to systematic risk, as the idiosyncratic of the individual security can be diversified away. This exposure is measured by the beta factor, which can be estimated by the following regression model. The excess return of the portfolio is regressed on the market risk premium (MRP):

$$r_{i,t} - r_{f,t} = \beta_i MRP_t + e_{i,t} \qquad (3)$$

The idea of Jensen (1968) is to allow for the existence of a non-zero constant. This constant is consequently called Jensen's alpha:

$$r_{i,t} - r_{f,t} = \propto_i + \beta_i MRP_t + e_{i,t} \qquad (4)$$

The intuition behind a positive alpha is that, after controlling for systematic risk, the portfolio is able to generate on average an additional return. In contrast to that, a negative alpha would reflect a portfolio that is inferior to the market portfolio.[21] However, since it's introduction the CAPM model has been tested extensively and several studies showed that the traditional CAPM model performs poorly in explaining cross-sectional return differences and that it might lag important risk factors (eg. Fama & French, 2004; Bodie *et al.*, 2014). In response several other models and extensions of the traditional model have been developed. The study makes use of an extension of the Carhart Four-Factor Model (Carhart, 1997), which builds on the Fama-French three factor model (Fama & French, 1993). The used model is similar to the specification in Fama and French (2012). It should better accommodate the variation in returns and make an alpha analysis more reliable, as many empirical tests showed that the additional loadings can explain much more of the variation in returns. The additional factors are size (SMB), book-to-market (HML) and one-year momentum (MOM).[22] These factors can be represented by the returns of value-weighted factor-mimicking portfolios, which lead to the following regression model:

[21] A similar approach (like in equation 4) is used by Ling and Naranjo (2002).

[22] At this point the study goes not into a discussion about the rationality of including the used risk factors. It rather relies on the documented empirical strength of the model to explain the variations in returns.

$$r_{i,t} - r_{f,t} = \propto_i + \beta_{1,i} MRP_t + \beta_{2,i} SMB_t + \beta_{3,i} HML_t + \beta_{4,i} MOM_t + e_{i,t} \quad (5)$$

Following the common praxis to solely choose a pure equity index as the market risk premium would not represent the opportunity set of an US mixed-asset investor. Thus, the market risk premium is divided into an equity market risk premium, a bond market risk premium and a commodity market risk premium. This makes it possible to differentiate between the exposures to the three different markets and better reflects the opportunity set of a mixed-asset investor.

One common problem of the risk measures discussed so far is that they assume normal distributed returns and do not give an impression of potential extreme losses in unlikely events hidden in the tails of the return distributions. This might lead to an underestimation of the true risk involved in an investment. The VaR measure partly cures this problem. It became famous in the late 80s and got its name through the RiskMetrics software solution (Longerstaey & Spencer, 1996). The measure is defined as the maximum loss over a given period within a confidence interval of a return distribution. It has to be noted that these risk measures are commonly expressed in absolute terms but that a representation in returns can be used as well. This leads to the following definition for discrete and continuous distributions:

$$\mathrm{VaR}_\alpha(R) = inf\{r|P[R \leq r] \geq \alpha\} \quad (6)$$

Thus, to find the VaR_α return, the α percentile of the return distribution has to be identified. The measure gives a good first impression of potential tail risk. However, it only measures the exact negative return at one particular confidence level and ignores the losses, which are less likely. This becomes especially problematic if the distribution of returns entails fat tails. The ES measure solves this problem as it reflects the average loss given that the return exceeds the specified VaR level (Acerbi & Tasche, 2002):[23]

$$\mathrm{ES}_\alpha(R) = E[R|R \leq \ \mathrm{VaR}_\alpha(R)] \quad (7)$$

Yamai and Yoshiba (2005) stress that the benefits of the ES measures do not come costless and that in cases of a fat-tail distribution or small sample sizes the estimation error of the risk measure is bigger than the error for the traditional VaR. They finally conclude that both methodologies should be seen as complementing measures and not competing ones. Therefore, both measures are applied to get a more comprehensive picture of the inherent risk.

Both metrics, the VaR and ES, can be calculated in different ways. The study uses two different ways. First, a non-parametric approach is used, which is solely based on the empirical distribution. Hence, it does not make any assumption about the return distribution. The problem is that the resulting values are less precise, especially with a limited sample size. This problem calls for parametric approaches, which try to estimate a continuous distribution function. Most

[23] The measure is also commonly known as “conditional VaR” or just “CVaR”.

commonly is the use of a simple gaussian distribution function, which is also used in the RiskMetrics tool (F. Modigliani & Modigliani, 1997). However, the use of a Gaussian distribution was critized for not taking higher moments like skewness and kurtosis into account, as commonly observed in financial time series. Hence, Zangari (1996) developed a method to adjust the downside risk measures by mean of a Cornish-Fisher expansion (Cornish & Fisher, 1938). Amédée-Manesme *et al.* (2015) also propose to use the suggested Cornish-Fisher expansion to correct the Gaussian VaR quantile z_α for skewness (S) and excess kurtosis (K) of real estate returns. The modified critical value can then be calculated by means of the second order expansion as follows:

$$\mathrm{cv} = z_\alpha + \frac{1}{6}(z_\alpha^2 - 1) * S + \frac{1}{24}(z_\alpha^3 - 3z_\alpha) * K - \frac{1}{36}(2z_\alpha^3 - 5z_\alpha) * S^2$$

$$S = E\left[\left(\frac{R-\mu}{\sigma}\right)^3\right] \tag{8}$$

$$K = E\left[\left(\frac{R-\mu}{\sigma}\right)^4\right] - 3$$

Boudt *et al.* (2008) expand this idea to the ES measure by using both Edgeworth and Cornish-Fisher expansions. It has been shown, that this parametric measures are more reliable estimates for VaR and ES when returns are not normally distributed (Amédée-Manesme *et al.*, 2015). Another advantage is the parsimonious in terms of computation resources compared to other approximations. Despite the advantages, it has to be mentioned that the used higher moments themselves are random variables and can be significantly biased for small samples. Furthermore, no trends in the metrics are considered and all observations are equally weighted (Zangari, 1996).

3.2 Covariance & Correlation Matrix

Correlation coefficients and the covariance matrix of a combination of assets are at the heart of risk diversification and portfolio management. Assuming that the real probability distribution of the respective security returns is known and that the correlations are below one, an investor can minimize uncertainty by putting different weights on the available assets (H. M. Markowitz, 1991). Hence, the objective is to specify the forecasted covariance matrix of the considered assets in the best possible way to be able to make superior asset allocation decisions subject to a required return or risk constraint (R. Engle & R. Colacito, 2006).

The simplest way to model the covariance matrix H is to assume a constant correlation matrix R and constant volatilities $\sqrt{h_i}$ on the diagonal of the D matrix. This leads to the following static model:

$$H = DRD, \qquad where\ D = diag\{\sqrt{h_i}\}. \tag{9}$$

The unconditional correlation coefficient can be stated as follows:

$$\rho_{AB} = \frac{E(r_A - \mu_A, r_B - \mu_B)}{\sqrt{E(r_A - \mu_A)^2 E(r_A - \mu_A)^2}}\ ; \tag{10}$$

However, assuming a constant correlation between two financial variables does not take time variation and different regimes into account. As indicated by the reviewed literature, it has been shown that correlation coefficients change considerably over time. Therefore, time-varying conditional coefficients in order to create better predictions and to get a better understanding of the time-varying diversification benefits of an asset should be used (Huang & Zhong, 2013). Hence, the goal is to fit a model, which allows a set of return series r_t to have a time-varying conditional covariance matrix H_t, given all available information Ω_{t-1} at the current point in time:

$$r_t|\Omega_{t-1} \sim N(\mu, H_t) \tag{11}$$

Rolling Correlations based on simple moving averages (SMAs) over a fixed window is one of the most common forms of conditional correlations and leads to a time dependent covariance matrix (Engle, 2002):

$$\rho_{AB,t} = \frac{\sum_{s=t-n-1}^{t-1}(r_{A,s} - \mu_{A,t})(r_{B,s} - \mu_{B,t})}{\sqrt{\left[\sum_{s=t-n-1}^{t-1}\left(r_{A,s} - \mu_{A,t}\right)^2\right]\left[\sum_{s=t-n-1}^{t-1}\left(r_{B,s} - \mu_{B,t}\right)^2\right]}} \tag{12}$$

Modeling correlations in this way has a few shortcomings as well. The most critical one is that the user has to specify a fixed window n. The outcome will extremely depend on the subjective choice of the size of the window. In particular, there is a trade off between a long window and a short window. The problem of a short window is that one time extreme events can lead to massive biases. The entry and also the dropout might cause extreme distortions, which are not justified. On the other hand longer windows might not sufficiently reflect recent market movements, as all observations are equally weighted (Alexander, 2008).

Some of these shortcomings were addressed by the introduction of exponentially smoothing and exponentially weighted moving averages (EWMA) for the estimation of a conditional correlation matrix. This method was endorsed by the RiskMetrics Group (Longerstaey & Spencer, 1996).

The basic idea is to put more weight on the most recent observations. It is implemented by using a geometrically declining weighting scheme. The user has to choose a factor between zero and one, which makes sure that older lagged values of the time series get less weight, as the factor shrinks and decays towards zero with the number of periods (Brooks, 2014). This becomes clear by looking at the following formula:

$$\rho_{AB,t} = \frac{\sum_{s=t}^{t-1} \lambda^{t-j-1} (r_{A,s} - \mu_{A,t})(r_{B,s} - \mu_{B,t})}{\sqrt{\left[\sum_{s=1}^{t-1} \lambda^{t-j-1} \left(r_{A,s} - \mu_{A,t}\right)^2\right] \left[\sum_{s=1}^{t-1} \lambda^{t-j-1} \left(r_{B,s} - \mu_{B,t}\right)^2\right]}} \qquad (13)$$

Moreover, it becomes obvious that the closer lambda is to one the more persistent is the correlation to a recent market shock. Thus, the correlation estimates would be much smoother in comparison to a low value of lambda, which would make the correlation efficient much more sensitive to occurring market shocks (Alexander, 2008). The modeling process entails the problem that the user has to specify the factor lambda. This study follows the suggestion of the RiskMetrics Group, who compared different decay factors by using a root mean squared error criterion and concluded that 0.97 for monthly data performs best (Longerstaey & Spencer, 1996).

For the following derivation of the more sophisticated dynamic conditional correlation (DCC) model, it is important to notice that the conditional correlation is equal to the conditional covariance between the standardized disturbances ε:

$$\rho_{AB,t} = \frac{E_{t-1}\left(\varepsilon_{A,t}\varepsilon_{B,t}\right)}{\sqrt{E_{t-1}(\varepsilon_A^2)E_{t-1}(\varepsilon_B^2)}} = E_{t-1}\left(\varepsilon_{A,t}\varepsilon_{B,t}\right) \qquad (14)$$

As the disturbance ε is standardized it has a mean of zero and a variance of one (Engle, 2002).
The DCC model is based on Bollerslev (1990) constant conditional correlation model, which assumes a constant correlation matrix R and only a time varying diagonal matrix D_t, containing the respective volatilities of the return time series:

$$\begin{aligned} H_t &= D_t R D_t, \qquad && where\ D_t = diag\{\sqrt{h_{i,t}}\}. \\ E_{t-1}(\varepsilon_t \varepsilon_t') &= R = D_t^{-1} H_t D_t^{-1} \qquad && since\ \varepsilon_t = D_t^{-1}(r_t - \mu) \end{aligned} \qquad (15)$$

In this model the univariate volatility dynamics might be modeled in different ways. However, the study concentrates on the dynamic conditional correlation model, as the use of dynamic conditional correlation (DCC) estimations is most appropriate when it comes to calculating the right covariance matrix for asset allocation purposes. This is documented by several studies, showing that DCC forecasts lead to superior investment decisions (e.g. Case *et al.*, 2012; R. F. Engle & R. Colacito, 2006; Peng & Schulz, 2013; Huang & Zhong, 2013). The model modifies the constant conditional model by additionally letting the correlation matrix vary over time, and thereby bringing univariate and multivariate dynamics together:

$$H_t = D_t R_t D_t. \tag{16}$$

The time-varying correlation matrix can be obtained by the following model (Hafner & Reznikova, 2012):

$$\begin{aligned} Q_t &= (1-\alpha-\beta)S + \alpha\varepsilon_{t-1}\varepsilon'_{t-1} + \beta Q_{t-1}, \\ R_t &= diag\{Q_t\}^{-1/2}\, Q_t\, diag\{Q_t\}^{-1/2} \end{aligned} \tag{17}$$

It can be estimated in different way. In this study a two step maximum likelihood estimation process based on Engle (2002) is used.[24] In a first step the univariate volatilities of the relevant return series are modeled by a ARMA(1,1)-GARCH(1,1) process. The second estimation makes then use of the obtained lagged standardized disturbances and estimates the parameters of the proxy process Q_t with S being the unconditional correlation matrix. Q_t might be interpreted as the dynamic conditional covariance matrix (Caporin & McAleer, 2013). The scalars α, β are strictly positive and the constraint $\alpha + \beta < 1$ is imposed. This ascertains that the process is stationary.

The use of this broad selection of different ways to express the correlation between two investments gives a sufficient and robust picture about the diversification benefits and potential changes over time.

3.3 Portfolio Optimization & Spanning Tests

The traditional mean-variance optimization, based on the groundbreaking work of H. Markowitz (1952), is applied for the portfolio optimization exercises. In order to calculate a portfolio on the mean-variance frontier the following minimization problem has to be solved:

$$\min_{w}\left[w'\mathrm{H}_t w - \frac{1}{A}w'R\right] \tag{18}$$

The first term represents the variance of the portfolio, which is calculated by multiplying the weight vector w with the covariance matrix H_t. The second term expresses the expected return of the portfolio divided by the level of risk-aversion A of the investor. Thus, varying the risk tolerance level can draw the efficient frontier. The higher the risk aversion of an investor is, the smaller the second term gets. This brings the investor closer to the minimum variance portfolio, which can be calculated by setting the second term to zero and minimizing the portfolio variance depending on the weights of the assets.

[24] The R package "rmgarch: Multivariate GARCH models." from Ghalanos (2015) was used for the computation process.

The mean-variance approach relies on several critical assumptions and is subject to the right choice of input factors. Chopra and Ziemba (1993) emphasize that errors in the mean estimations can lead to inferior allocations. They point out that the optimal asset allocation is more sensitive to the right choice of means than to variances and covariances. For Black and Litterman (1991) the only meaningful approach is to start from equilibrium returns, which can be reverse engineered by using the current market weights of the used assets. However, this approach is not appropriate, as the market for listed real estate is still in its beginning and relative weights might not sufficiently represent the “true” market equilibrium weights. Thus, implied returns and current market weights should at most be seen as a vague orientation and not as adequate for this study. Therefore, the study discusses the mean-variance frontiers, which rely on the mean returns of the whole sample period, as longer time series data should yield more reliant means (D. Brounen & Eichholtz, 2003). Only for the time-varying tests, the calculation time frames are shorter, as the available data for the portfolio manager is limited in a more realistic way.[25] Another critical assumption is the reliance on normal distributed returns. Thus, the traditional mean-variance optimization does not take tail risk as deviations from normality into account. This can lead to a flawed asset allocation. Therefore, investors have to keep the analysis of the distribution curves and the risk measures for tail risk in mind before drawing to early conclusions about the results of the optimization exercises.

The formed mixed-asset benchmark and test portfolios are additionally tested for statistical difference by means of mean-variance spanning tests. The tests were developed by Huberman and Kandel (1987) to analyze the effect of additional assets on a benchmark portfolio. If the compared frontiers have one particular point in common and the frontiers intersect, one speaks consequently about intersection. Hence, there has to be one utility function for which, adding the chosen additional assets does not make a difference. Another scenario is the case of coinciding frontiers. Such a scenario is called spanning, as no mean-variance investor, no matter what risk-aversion level, can benefit from extending the portfolio. The problem is that in almost every case of a finite sample the estimated frontier will shift by adding additional assets (Nijman & DeRoon, 2001). However, the question is if the shift is truly significant or is due to an estimation error. In order to answer this question, tests for significance are required. The study applies regression-based tests for several portfolios along the whole mean-variance frontiers. This yields a clear and comprehensive picture in which areas of risk extending a portfolio might be significant.

The approach consists of four simple steps. First, the weight vectors for both portfolios are obtained from the optimization exercise. Second, the weights are used to calculate the realized returns $r_{P,t}$ of the portfolios. This can be done by multiplying the obtained transposed optimal weight vectors w with the historical return vectors $r_{A,t}$ of the relevant assets:

$$r_{P,t} = w' r_{A,t} \tag{19}$$

[25] Nevertheless, the website enables the reader to limit the time frame to get an impression about the sensibility of the gap between the frontiers with and without real estate.

Both, the weight vectors and the return vectors have the dimensions Nx1, where N stands for the number of assets. Third, the calculated returns are used to run a regression of the following form:

$$r_{N,t} = \alpha + \beta r_{B,t} + \varepsilon_t. \tag{20}$$

The return r_N stands for the realized returns of the test portfolio with the additional assets included. Consequently, the return r_B stands for the returns of the benchmark portfolio. Fourth, test the null hypothesis:

$$H_0\colon \alpha = 0,\ \delta = 0\ , \quad \text{with} \quad \delta = 1 - \beta. \tag{21}$$

The null hypothesis is very intuitive and states that the returns of the benchmark portfolio can on average replicate the newly formed portfolio and that the additional assets cannot on average generate a superior return at the same level of systematic risk. Hence, the benchmark portfolio would dominate the test portfolio, as adding the additional assets would only increase the unsystematic risk in form of the SD of the error term in equation 20. In this case, an extension could not increase the return-risk ratio (Nijman & DeRoon, 2001).

3.4 Tests for Inflation Hedging

The possibility to use real estate as an inflation hedge for an US investor is analyzed with different regression-based approaches. The short-term analysis is based on the traditional Fama and Schwert (1977) equation and an extended version, which controls for economic variables. For the long-term analysis an error correction model is used to get an understanding of the short-term dynamics related to the long-term equilibrium. The whole analysis is solely conducted from the perspective of an US investor.

Following the most cited version of the Fisher equation (Fisher, 1930),

$$E(r_{t+1}) = E(\gamma_{t+1}) + E(\pi_{t+1})\ , \tag{22}$$

the expected nominal return $E(r_{t+1})$ should equal the sum of the expected real rate $E(\gamma_{t+1})$ and the expected inflation rate $E(\pi_{t+1})$. If it is assumed that the inflation rate and the real rate move independently, the expected return and the expected inflation rate on risky assets should move in lockstep. Based on this assumption, it is possible to test the fisher hypothesis with the following regression model, which analyzes the contemporaneous relation between inflation rates and the returns of an asset:

$$r_t = \alpha + \beta \pi_t + \varepsilon_t\ . \tag{23}$$

If the fisher equation is right, the null hypothesis, that is $H_0\colon \beta = 1$, should hold (Fama & Schwert, 1977). However, the expected inflation rate does not equal the realized inflation rate as the realized rate is accompanied by noise. This unexpected inflation component induces a potential measurement error in model 23. Thus, the realized inflation rate should be divided in expected inflation (EI) and unexpected inflation rate (UI) to individually evaluate both impacts on asset returns. This leads to the Fama and Schwert (1977) regression (Hoesli *et al.*, 2008):

$$r_t = \alpha + \beta E\mathrm{I}_t + \gamma UI_t + \varepsilon_t \tag{24}$$

EI_t describes the expected inflation rate conditional on the given information one period ago and UI_t is the realized unexpected inflation. The unexpected inflation rate is simply calculated as the difference of the total realized inflation rate and the expected inflation rate. The clear advantage of this model compared to equation 23 is the possibility to distinguish between unexpected and expected inflation hedging capabilities. If an asset is a complete hedge, both β and γ should equal one (Fama & Schwert, 1977). The main caveat of the model is the estimation of the expected inflation. In this context several suggestions have been made. The most common solution is based on the assumption that the expected return on a treasury bill can be decomposed into the expected real rate and the expected inflation rate (Hoesli *et al.*, 2008). This idea is based on the assumption that the return of a treasury bill can be seen as a quasi risk-free investment only depending on inflation risk. This study follows the suggestion of Kaul (1987), and Liu *et al.* (1997) to use the model of Fama and Gibbons (1984):

$$EI_t = TB_{t-1} - \frac{1}{12}\sum_{s=t-1}^{t-12}[TB_{s-1} - CPI_s] \tag{25}$$

The model calculates expected inflation (EI_t) as the difference of the one period lagged treasury rate and the expected real rate. The expected real rate is represented by a twelve-month SMA of the difference of lagged Treasury bill rate and inflation rate. Treasury bill rates are lagged one period, as they are forward-looking interest rates based on the most recent auctions. Further smoothing methods, like ARMA and MA processes, have been suggested. For example, Hoesli *et al.* (2008) use a MA(4) process to calculate expected inflation.[26] Another approach is to disentangle the total inflation by means of an errors-in-variables model (e.g. Dirk Brounen *et al.*, 2014 and Boudoukh & Richardson, 1993). For this purpose a two-stage least square approach can be used. The first stage consists of estimating a regression model for the expected inflation E(I) by choosing a set of instruments. The instruments should be chosen in a way that they are correlated with expected but are uncorrelated with unexpected inflation. Short-term interest rates and lagged values of the inflation rate are an obvious choice and were used in several studies. In the second step the predictions and the residuals of the model can be used to run the regression of form 24. Alternatively, a generalized method of moments approach (IV-GMM) can be used. This approach has the advantage that no assumption about the distribution of the error terms has to be made and the model can be estimated robust against heteroscedasticity. This comes with the cost that the model could yield inferior results if used with a small sample. Nevertheless, the

[26] Tests with a MA(4) to calculate expected inflation yield similar results but are not displayed.

advantage of using instruments is that no explicit model for the expected inflation rate has to be specified (Dirk Brounen *et al.*, 2014). However, the choice of the right instruments can be seen as a similar problem. Despite this options the study solely presents the results based on expected inflation calculated by formula 25, as it is a very accessible, intuitive, and transparent method to calculate expected inflation.

In addition to the pure short-term analysis, an error correction model is used. The idea of the model is to allow for an error correction term (ECT) in a short-term model of first order integrated variables. Thus, the model comprises both long-term and short-term relation and can describe how a potential long-term equilibrium is achieved (Brooks, 2014). In a first step, the long run relationship has to be specified in form of a regression model. For this study a multivariate model of the following form is used:

$$R_t = \beta_0 + \sum_{i=1}^{n} \beta_i X_{it} \quad . \tag{26}$$

R_t represents the level of the real estate index and X_{it} a vector containing expected, unexpected inflation indices or just the realized inflation. Moreover, other economical control variables are included. The inflation indices are formed with the estimated expected, unexpected inflation and realized inflation rates. Furthermore, all variables are scaled and logged to avoid scaling effects and to make the interpretation of first differences as continuous returns more intuitive. The decision to include further explanatory variables is due to the fact that several studies pointed out, as discussed above, that an exclusion of different economic fundamental variables leads to an omitted variable bias. The selection of the explanatory variables is based on the study of Hoesli *et al.*, 2008 and backed by the observations of further previous studies and the idea of the proxy hypothesis (e.g. Glascock *et al.*, 2002; Fama, 1981). Before starting with the regression analysis, each time series is checked for first order integration to get a better picture of the reliability of the tests. This is done by means of ADF tests and Phillips-Perron (PP) tests (e.g. see Brooks, 2014).[27] In the second step, an ADF test on the obtained residuals of the model is applied to test for cointegration. If the null hypothesis of the unit root is rejected, the error term will be stationary and will fluctuate around zero. Then, investors can assume that the index and the significant explanatory variables are in an equilibrium relationship (Verbeek, 2012).[28] In this case, the residuals of the level regression can be used as an ECT in the following model of first differences:

$$\Delta R_t = \alpha_0 + \sum_{i=1}^{n} \alpha_i \Delta X_{it} - \gamma u_{t-1} \tag{27}$$

[27] Mainly the R package „urca“ of Pfaff and Stigler (2013) is used for the calculations. Originally, the ADF test goes back to Fuller (1976) and Said and Dickey (1984). The PP test was developed by Phillips and Perron (1988).
[28] The outlined test is also well-known as the Engle-Granger two-step cointegration test (Engle & Granger, 1987).

In the stated model, the change of the index depends on the continuous returns of the same explanatory variables and additionally on the lagged residuals u_{t-1} obtained from the long-term level regression. The intuitive idea is that the negative ECT indicates how quickly the variables would find their way back to the equilibrium relation, if there has been a deviation from the equilibrium in the previous period. A coefficient of one implies immediate adjustment, whereas a coefficient close to zero indicates a slow adjustment process.

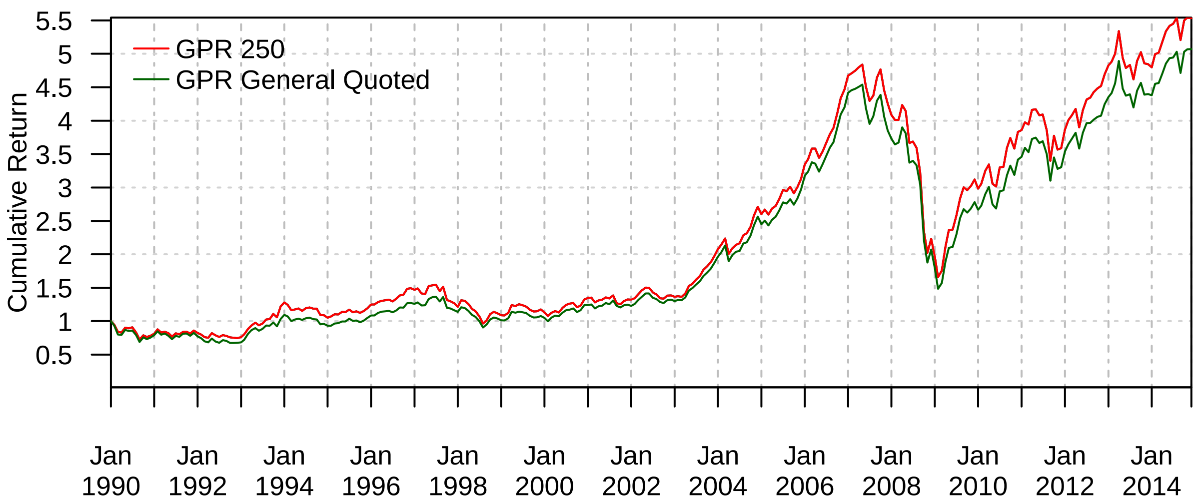
The introduced methodologies form a reliable framework for a comprehensive inspection of the three outlined subquestions. However, in order to get sound results the right input factors and data play an equally important role. Thus, the next chapter discusses the choice of data and provides detailed information about the used data.

4 Data

Listed Equity Real Estate

The indices provided by Global Property Research (GPR) are used, in order to approximate the international markets for listed equity real estate. These indices partly date back to 1983. The user can also conduct part of the tests with the FTSE EPRA/NAREIT Indices online but results are not explicitly displayed in this paper. However, as the correlations of monthly returns between the indices and subindices are almost exclusively above 0.9 and descriptive statistics are similar (see Appendix G), similar results are expected. Furthermore, both the GPR General Quoted index and the FTSE EPRA/NAREIT Index exclude open-ended funds to solely represent real estate companies.

Fig. 3:
Cumulative Returns – GPR Indices



Notes: The figure plots the cumulative monthly total returns of the global GPR indices. The sample period ranges from 1990:1 to 2014:12.

The decision to primarily use the GPR indices is backed by Serrano and Hoesli (2008), who compare different benchmarks for the worldwide performance of the market for real estate securities. They conclude, that the GPR General Index and the S&P Citigroup World Property Index are most appropriate to evaluate the performance of the market as a whole, as they are the broadest available global indices. However, they see the GPR 250 and the FTSE EPRA/NAREIT Global Real Estate Index as more replicable for investors, as both indices take liquidity of their constituents into account. For instance, a company needs a free float MCAP over $50m to qualify as a constituent in the GPR 250, whereas only an overall MCAP bigger than $50m is needed to enter the GPR General Quoted. Furthermore, an operative turnover over 75 percent from real estate related activities is an important requirement for both GPR indices. Moreover, both GPR indices are dominated by US companies with weights of around 55 percent for the GPR 250 and 44 percent for the GPR General Quoted according to the latest quarterly review of GPR. The higher weight of the USA companies in the GPR 250 leads to an even more dominant role of REITs, with 81% (75% for the GPR General Quoted), over REOCs.[29] Further details about the indices are provided by Serrano and Hoesli (2008) in Exhibit 9 and can be found on the providers websites.

Despite the differences in index construction, inclusion criteria and compositions both GPR indices depict a similar development (Fig. 3). Moreover, the monthly returns are highly correlated with a coefficient of 0.988. This suggests that the choice between the two indices should not yield different results. Nevertheless, the GPR General Quoted is mainly used as it contains more companies and returns should be a more reliable representation of the global real estate markets in early years.

GPR and also FTSE offer country and regional sub-indices in US dollars and local currencies. This makes it possible to analyze the performance of national real estate markets separately and to perform more detailed portfolio optimization exercises. Despite the availability of local currencies only the results for an unhedged US investor are displayed.[30] The country indices are chosen based on data availability, development and transparency of the local real estate market. All considered real estate markets count to the most transparent and developed markets in the world and should allow a reliable analysis (Feenan *et al.*, 2014). Nevertheless, it should be highlighted that some of the country's indices only consisted of a limited number of constituents in their early years. Hence, values were more prone to misvaluations.

Fixed Income

As a proxy for the global bond market, the Barclays Global Aggregate bond index, which dates back to 1990, is used. The index measures global investment grade debt (no rating below Baa) from twenty-four different local currencies. It considers both developed and emerging market issuers. The benchmark covers fixed-rate treasury, government-related, corporate and securitized bonds. Fixed-rate treasury bonds represent the biggest part of the benchmark with around 54 percent. For the empirical tests unhedged monthly and quarterly total returns are used.[31]

For the country analysis the subindices of the world government bond index (WGBI) from Citigroup are used. The index does not cover the scope of bonds like the Barclays Global Aggregate index, but offers country subindices, mainly dating back to 1985. The exceptions are Hong Kong, Sweden and Singapore. Hong Kong does not have a government bond market and government bonds were not issued for a big part of the sample in Singapore. For Sweden, the government bond data is limited. However, the countries are included in most parts of the analysis, as they represent well developed listed real estate markets with high MCAPs (Kroencke & Schindler, 2012). The limitation to government bonds should still sufficiently reflect the development of the local bond markets, as the biggest part of the global bond markets consists of government related securities.

[29] This is due to the fact that the American subindex almost solely contains REITs (97%). The percentages between REITs and REOCs within the African and European subindices are much more balanced.
[30] The reader can additionally choose local currencies (assuming a perfect currency hedge) on the provided website.
[31] The Barclays Multiverse further includes high-yield bonds below investment grade but only dates back to 1999.

It is important to get an impression of the interest rate sensitivity of the underlying indices to better evaluate the strength of the price effect component of the total return.[32] Based on the information of the latest factsheets, the durations of the Barclays Indices and the Citigroup Indices are lying between six and eight years, indicating strong price effects due to interest changes.

Equity

The MSCI All Country World Index (ACWI) will be used as a proxy for the worldwide equity market if possible. The index brings the MSCI World and the MSCI Emerging markets together and therefore even better represents the global equity market. However, historical data dates only back to 1988. Thus, for the inflation analysis the MSCI World with its longer data availability is used instead. MSCI also provides broad country equity subindices, which should properly reflect the performance of the local stock markets. Times series data from 1970 onwards is available for all countries.

Table 1:
Real Estate Exposure of Equity Indices

Index	**MSCI Equity** (USD mio)	**MSCI Real Estate** (USD mio)	**Exp.** (%)	**GPR General Quoted** (USD mio)	**Exp.** (%)
All Countries	38,526,172	1,193,996	3.10	1,251,482	3.25
Developed Countries	34,327,890	1,080,983	3.15		
Europe	8,807,254	117,946	1.34	286,735	3.26
Asia Pacific	7,488,185	461,784	6.17	362,742	4.84
Australia*	966,092	78,780	8.16	57,676	5.97
Canada	1,318,212	38,301	2.91	52,139	3.96
France*	1,327,315	45,409	3.42	64,802	4.88
Germany*	1,232,612	19,249	3.68	39,856	3.23
Hong Kong*	445,563	135,257	30.35	90,686	20.35
Japan*	2,997,691	136,919	4.57	123,332	4.11
Netherlands	367,091	-	-	-	-
Singapore*	265,471	33,433	12.59	58,039	21.86
Sweden	404,649	-	-	19,498	4.82
USA	19,495,784	675,214	3.46	658,709	3.38
UK*	2,740,503	45,374	0.70	99,497	3.63

Notes: Dividing the MCAP of the respective equity index by the appropriate real estate MCAP approximates the real estate exposure. The asterisk indicates that the MCAP is calculated based on the weights of the regional MSCI indices. The data is from MSCI and GPR factsheets of the individual indices from April 2015.

Most of the equity indices already contain listed equity real estate companies. In some countries the companies account for high percentages of the total stock market. This weakens the validity of the empirical results. Therefore, the exposure has to be considered when comparing both types

[32] Spierdijk and Umar (2015) give a good summary in Appendix A1 about the common calculation and the different components of the total return of bond indices.

of investment as distinct asset classes. Only then reliable interpretations of the results and portfolio weights are possible (Idzorek *et al.*, 2006). Based on the latest factsheets of MSCI and GPR, the study tries to give a vague impression of the real estate exposure within the local equity market. Table 1 shows the approximate exposure to listed equity real estate. The exposure is below five percent for most local and regional markets. For those markets, the impact on performance and risk measures should be imperceptible. However, in the Asian pacific region, the influence is stronger, mainly because of the high exposure for Hong Kong and Singapore. As the market for real estate companies is growing and the percentages will very likely get higher in the future, indices explicitly excluding real estate companies are becoming more desirable.[33]

Commodities

As an additional alternative asset class, the study includes a global commodity index, as the interest of portfolio managers in commodities is constantly growing and therefore should be considered in the optimization process. In particular, when analyzing real estate, which should be closely related to the underlying property market, the inclusion of real assets in form of commodities is of high interest. Furthermore, both asset classes should be closely related to inflation, which should lead to similar results for the inflation analysis. If this is the case one asset might be redundant for inflation hedging purposes. The S&P Goldman Sachs Commodity Total Return Index (GSCI), which mainly consists of oil related instruments (over 50 percent), is used. The choice is backed by Chong *et al.* (2009), who use the index for their correlation analysis as well. The available time series data dates back to 1969.

Inflation Hedging

For analyzing the inflation hedging capabilities of the asset classes, the Consumer Price Index for All Urban consumers (in the following only CPI) is used. The quarterly data of the consumer price index for the US is not seasonally adjusted, as it would make the separation of unexpected and expected inflation more difficult due to lost detail.[34] Furthermore, it is important to pay attention to the composition of the index, as an insight should improve the quality of the interpretations of statistical results. The headline sector housing is of the highest interest, as one would expect a connection between costs of private housing and commercial rents. Based on the latest weights from the bureau of labor statistics (BLS), housing has a percentage of over 42.2%. The most intuitively and closest relation should be between commercial rents and the two subcategories “Rent of primary residence” (7.2%) and “Owners’ equivalent rent of primary residence” (24.3%). Therefore, it should be of additional interest to test the hedging capabilities of real estate related to the mentioned subindices of the CPI. The results of the ECM for the subindices are not discussed in the main part but can be found in Appendix E. For the analysis further economical variables are used as control variables. The variables are chosen based on the work of Hoesli *et al.* (2008) and other previous literature about inflation hedging, which were discussed in the literature section. A detailed overview is given in Appendix A.

[33] With the introduction of real estate as a new headline sector in the GICS, hopes for such indices become more tangible.
[34] The BLS mainly uses ARIMA models to make seasonal adjustments.

Other Data

Furthermore, two benchmarks are constructed to compare the out-of-sample performance of optimized portfolios. Two different constructions are used consisting of equity and bonds. The first version is represented by the common construction of a portfolio consisting of 60 percent equity and 40 percent bonds. The second version is more sophisticated and builds on the idea that the market is efficient and chooses weights in a consistent way. The monthly market values of the S&P 500 Composite and the Barclays US Aggregate are used to approximate the market weights for equity and bonds. The return of the current period is subsequently calculated by multiplying the one period lagged weights by the realized returns of the indices.

Moreover, the risk loadings for the alpha analysis are taken from Kenneth R. French website at Dartmouth. For national investors the MSCI USA and the Barclays US Aggregate are used to calculate the respective market risk premiums. For global investors, the MSCI ACWI and the Barclays Global Aggregate are used. Kenneth R. French website also offers risk factors based on global replicating portfolios. However, those are not available for the whole considered time period. Therefore, the US risk factors are used as an approximation.

A detailed overview of providers, vendors and data availability is provided in Appendix A. The outlined methodologies in combination with the presented data lead to the empirical results in the next chapter.

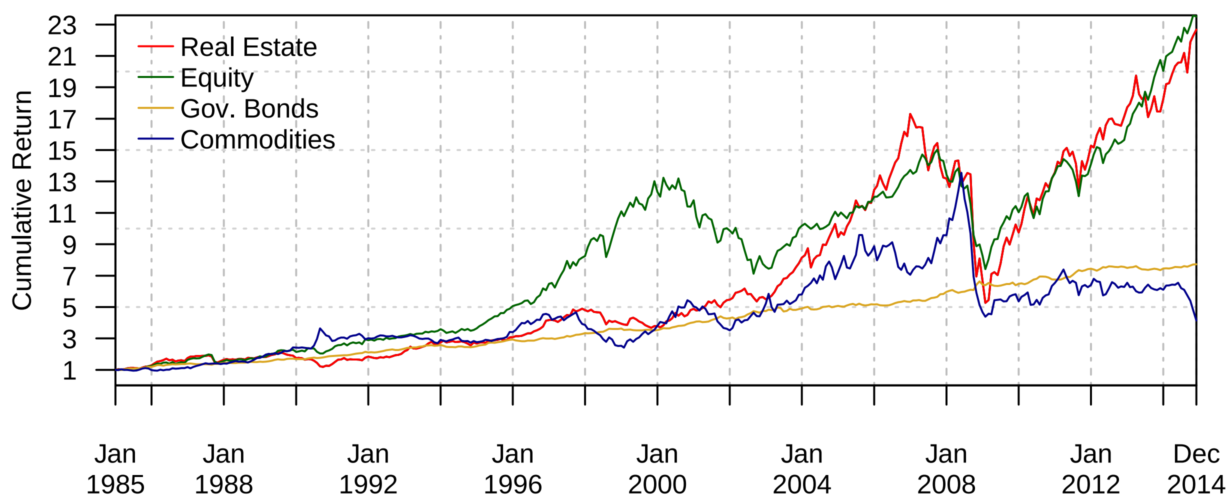
5 Empirical Findings

5.1 National Portfolio

5.1.1 Return Enhancement

In order to evaluate the return enhancement capabilities of national real estate within a pure US portfolio, investors have to get an impression of the risk and the performance of an investment. Fig. 4 depicts that real estate performed differently compared to equity up to the beginning of the financial crisis. In particular, between 1997 and 2003 the biggest deviation of the two indices can be observed. The gap can be explained by the occurrence of the dotcom bubble, which has not affected the real estate market in a visible manner. Nevertheless, it can further be observed that both markets reacted similar to the financial crisis and that the recovery phase followed closely related patterns as well. Thus, the first impression suggests time-varying return enhancement capabilities.

Fig. 4:
National – Cumulative Returns



Notes: The chart illustrates the cumulative monthly total returns of the four different asset classes in the USA. The base year is 1985.

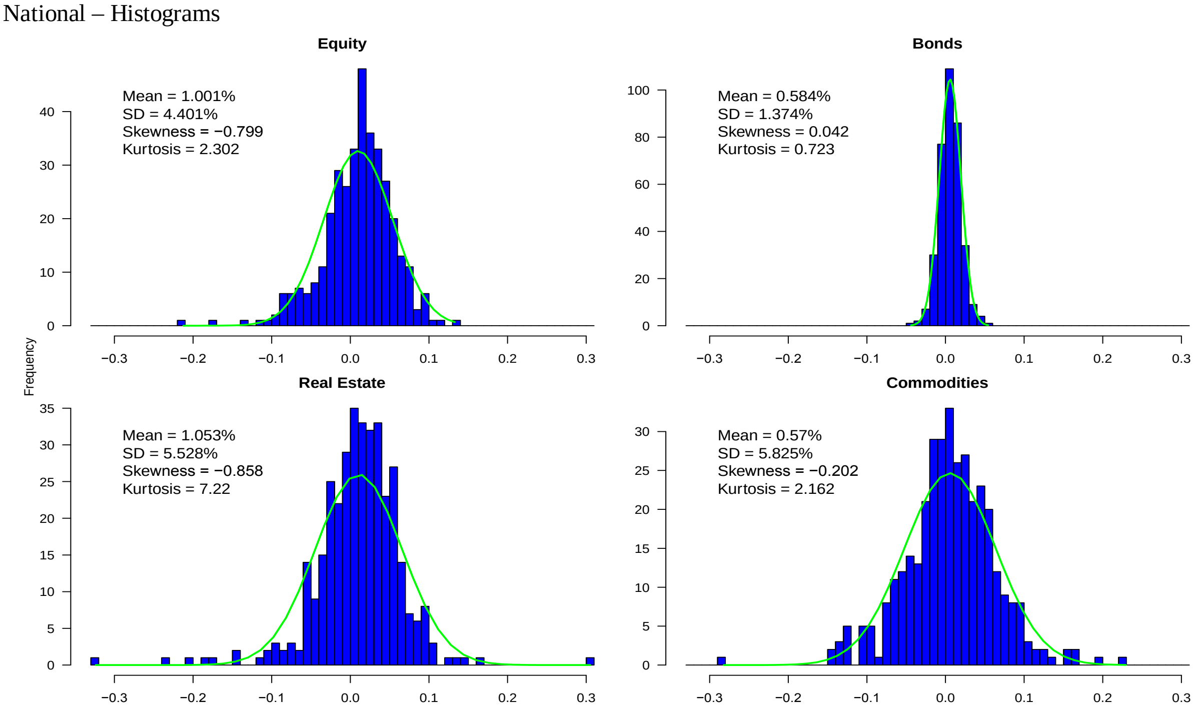
This idea is further emphasized by the descriptive statistics for the three different time periods in Table 2 and the time-variation in mean returns in Fig. 6. Real estate has the highest mean return over the whole sample but is closely followed by equity. At the same time it displays a higher SD of monthly returns. Only the commodity investment had an even higher SD. Both the mean and the return-risk ratio of real estate for the subsample from 1985:1 to 1999:12 are considerably lower compared to equity. In the second displayed subsample the picture is reversed. Real estate has the highest mean return and the second highest return-risk ratio during the period from 2000:1 – 2014:12. Nevertheless, real estate returns show a very high SD in the period and the return-risk ratio of government bonds is still significantly higher. The high SD during this period might be explained by the slump and the strong recovery of real estate prices due to the financial crisis in 2008.

Table 2:
National – Descriptive Statistics of monthly Returns

	1985:1 – 2014:12			1985:1 – 1999:12			2000:1 – 2014:12		
	Mean	**SD**	**Ratio**	**Mean**	**SD**	**Ratio**	**Mean**	**SD**	**Ratio**
Equity	1.001	4.401	0.227	1.574	4.306	0.366	0.428	4.432	0.097
Gov. Bonds	0.584	1.374	0.425	0.722	1.422	0.508	0.445	1.313	0.339
Real Estate	1.053	5.528	0.191	0.888	4.413	0.201	1.218	6.462	0.189
Commodities	0.570	5.825	0.098	0.822	4.721	0.174	0.319	6.755	0.047

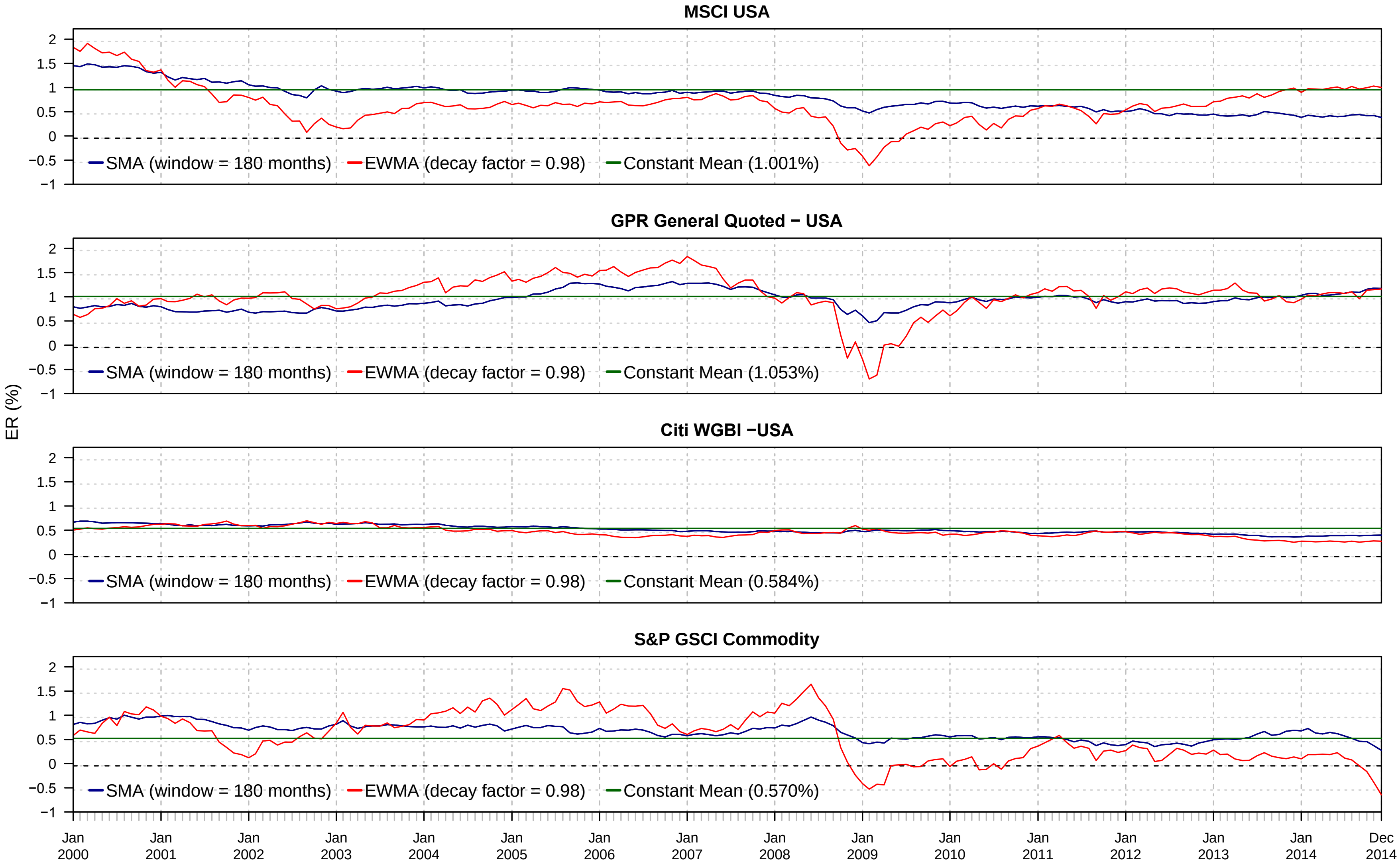
The extraordinary recovery phase of real estate can also be seen in Fig. 6. The SMA and EWMA of real estate returns quickly move back to the constant mean. In contrast, the indicators for equity move back slower towards their constant mean. The SMA of equity even decreases.
The observed difficulty to evaluate the return enhancing capabilities of a real estate investment is mainly due to the two extreme events in form of the dotcom bubble and the financial crisis during the sample period. The two events were characterized by extraordinary high returns asking for risk metrics, which further evaluate the possibility of tail risk. Fig. 5 shows that the monthly return distributions deviate from normal distributions. In particular, the distribution of real estate has the highest negative skewness and kurtosis. Furthermore, more negative extreme returns, compared to the other asset classes, can be identified.

Fig. 5:
National – Histograms



Notes: The figure depicts the histograms of monthly returns over the whole sample period (1985:1-2014:12). The returns are summed in one percent buckets. In order to make the histograms easier to interpret the normal distribution curve is added in green.

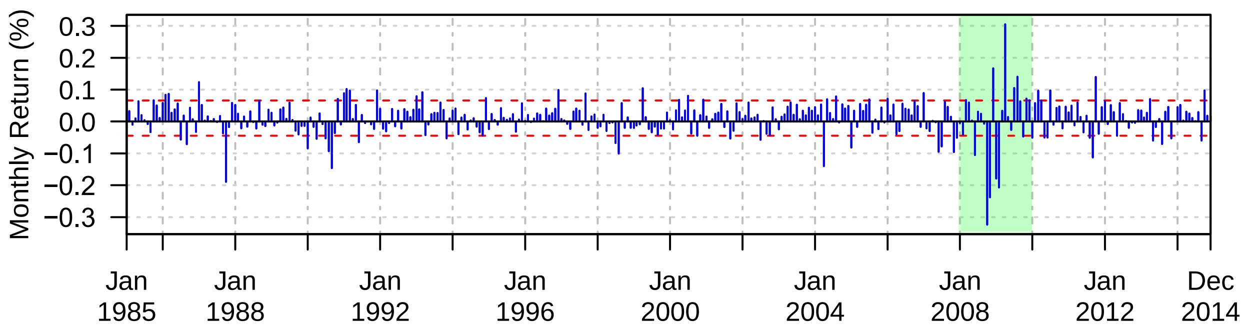
Fig. 6:
National – Time-varying Mean Returns



Notes: The graph depicts mean returns based on different calculation methods. Beside the constant unconditional mean, a SMA with a window of 180 months and an EWMA with a decay factor of 0.98 is displayed. Monthly return data from 1985:1 to 2014:12 is used.

As expected, most of these extreme returns fall in the period around the financial crisis, as indicated by the highlighted area in Fig. 7. Moreover, the maximum drawdown of real estate, with around 70 percent, was almost 20 percent higher compared to equity during this time period. This observation may question a general fat tailed distribution of returns, as one might consider the crisis as a one-time event. On the other hand, there might exist boom and bust cycles in real estate markets. However, an exclusion or trimming of the displayed returns would bias the picture of the downside risk involved in an investment. Therefore, returns are not adjusted in any form.

Fig. 7:
National – Monthly Returns of Real Estate over Time



Notes: The graph depicts the monthly returns of the GPR General Quoted US subindex over the sample period 1985:1 to 2014:12. The red dotted lines set the boundaries of the area, which is within one SD away from the mean.

The deviation from normality and the identified extreme returns call for risk measures like VaR and ES. Table 3 strengthens the picture and gives an economical interpretation about the potential loss in the case of the returns within and at the lower five percent percentile. VaR is highest for a commodity investment followed by real estate and equity depending on the calculation method. Based on the Cornish fisher approximation the VaR, with a negative return of 8.5 percent monthly and 14.7 percent quarterly, is significantly higher for real estate compared to an equity investment. However, when the empirical calculation is used, the VaR is considerably lower expressing the difficulty to approximate the true return distributions and highlighting the limited sample size.[35] Furthermore, a real estate investment displays the highest expected average loss in 5 out of 100 month. Based on the empirical calculation only the ES of a commodity investment is higher for quarterly returns. It should be highlighted that the ES based on the Cornish fisher approximation is, with around 19.6 percent, more than 8 percent higher for a real estate investment compared to an equity investment based on monthly returns. On a quarterly basis the difference is even higher with over ten percent.

[35] Thus, quarterly values should be even more biased compared to monthly measure.

Table 3:
National – VaR and ES

	Equity	Gov. Bonds	Real Estate	Commodities
ES Monthly				
Empirical	-10.003	-2.350	-13.394	-12.913
Cornish Fisher	-11.534	-2.342	-19.599	-13.986
ES Quarterly				
Empirical	-17.528	-2.787	-23.674	-25.078
Cornish Fisher	-17.358	-2.968	-27.932	-26.356
VaR Monthly				
Empirical	-7.189	-1.722	-6.723	-9.411
Cornish Fisher	-6.970	-1.636	-8.491	-9.071
VaR Quarterly				
Empirical	-13.502	-2.113	-11.324	-13.654
Cornish Fisher	-11.818	-2.251	-14.745	-16.415

Notes: The table displays VaR and ES below the five percent quantile. The calculations are based on monthly and quarterly returns of the sample period 1985:1 to 2014:12.

Further tests in form of a multi-factor model for the three different time periods do not shed light in the dark. For the time period from 1985:1 to 1999:12 a negative alpha, which is significant at a 10 percent level, can be identified. Nevertheless, it is interesting to see that in contrast to the high SD of real estate the highly significant coefficient for the equity and bond indices are mainly below one.[36] In combination with the relatively low R squared it seems like an omitted risk factor is additionally driving real estate returns and might explain the high tail risk and volatility of real estate investments.[37] This would already imply room for diversification purposes, as the investor would be able to get exposure to different risk factors.

Table 4:
National – Multi-Factor Model Regressions

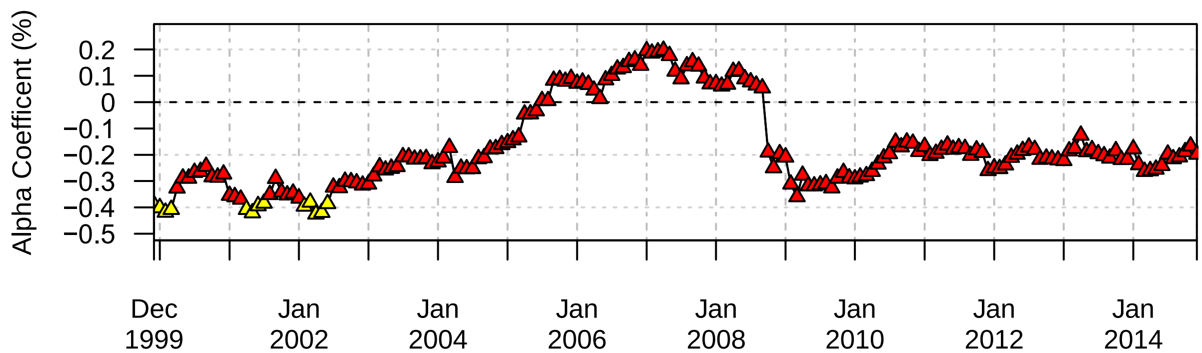
	1985:1-2014:12			1985:1-1999:12			2000:1-2014:12		
	Coeff.	Prob.		Coeff.	Prob.		Coeff.	Prob.	
Alpha	-0.002	0.389		-0.004	0.092	*	-0.002	0.559	
MSCI USA	0.815	0.000	***	0.788	0.000	***	0.805	0.000	***
Barclays US Aggregate	0.742	0.000	***	0.389	0.021	**	1.383	0.000	***
S&P Commodities	-0.041	0.335		-0.068	0.169		-0.028	0.582	
SMB	0.503	0.000	***	0.521	0.000	***	0.482	0.000	***
HML	0.769	0.000	***	0.542	0.000	***	0.897	0.000	***
MOM	-0.115	0.033	**	-0.013	0.851		-0.142	0.031	**
Adj. R squared	0.604			0.627			0.619		

Notes: The table presents the regression results of the Multi-Factor model (equation five). The monthly return of the GPR General Quoted US is the dependent variable. Newey-West standard errors are used to control for autocorrelation and heteroscedasticity. The significance of the coefficients at different confidence level is indicated by the number of asterisks: * 10%, ** 5%, and *** 1%.

[36] A conducted rolling beta analysis based on the specification in Table 4 yields similar results. Nevertheless, stronger reactions to varations of the bond index can be observed after the financial crisis.
[37] Business cycle risk should already be covered by the market risk loadings.

The picture does not change for different time periods based on a rolling alpha analysis, as shown in Fig. 8. Like in Table 3 only weakly significant and negative alphas can be identified for time periods, not fully including the area of the dotcom bubble. This indicates a worse performance of around 40 basis points per month without having lower risk in time periods before 2003. From an economical standpoint, this has to be considered as a highly significant difference. A possible explanation might be the bad performance of listed real estate due to the savings and loan crisis, overbuilding, and capital shortage during the late 1980's (S. Lee, 2010). Furthermore, the strong performance of the equity market during the technology boom should have played its part.

Fig. 8:
National – Alphas of Real Estate based on rolling Regressions



Notes: The graph depicts the alphas and their significance of the rolling multi-factor model over a window of 15 years. The regression specification is like in Table 4. Red dots stand for insignificant alphas, yellow dots for significant at the ten percent level and green dots for significant at the 5 percent level.

To sum up, the results are mixed and no clear evidence can be found that national real estate can be used as a return enhancer for a portfolio already consisting of equity, bonds and commodities. In particular the identified high tail risk and negative alphas question the inclusion in order to improve the risk-adjusted return of a portfolio. Considering the slightly higher return compared to equity over the whole sample period and several sub periods an investor may consider real estate as an absolute return enhancer. However, the economical significance with a difference in means of only five basis points over the whole sample period is rather weak.

5.1.2 Diversification Benefits

Table 5 shows the unconditional correlation matrix for the whole sample period. The low correlation of real estate with the three asset classes would justify an inclusion of real estate for diversification purposes at first sight.

Table 5:
National – Constant Correlations

	Real Estate	Equity	Gov. Bonds	Commodities
Real Estate	1.000	0.618	0.047	0.096
Equity	0.618	1.000	-0.003	0.148
Gov. Bonds	0.047	-0.003	1.000	-0.075
Commodities	0.096	0.148	-0.075	1.000

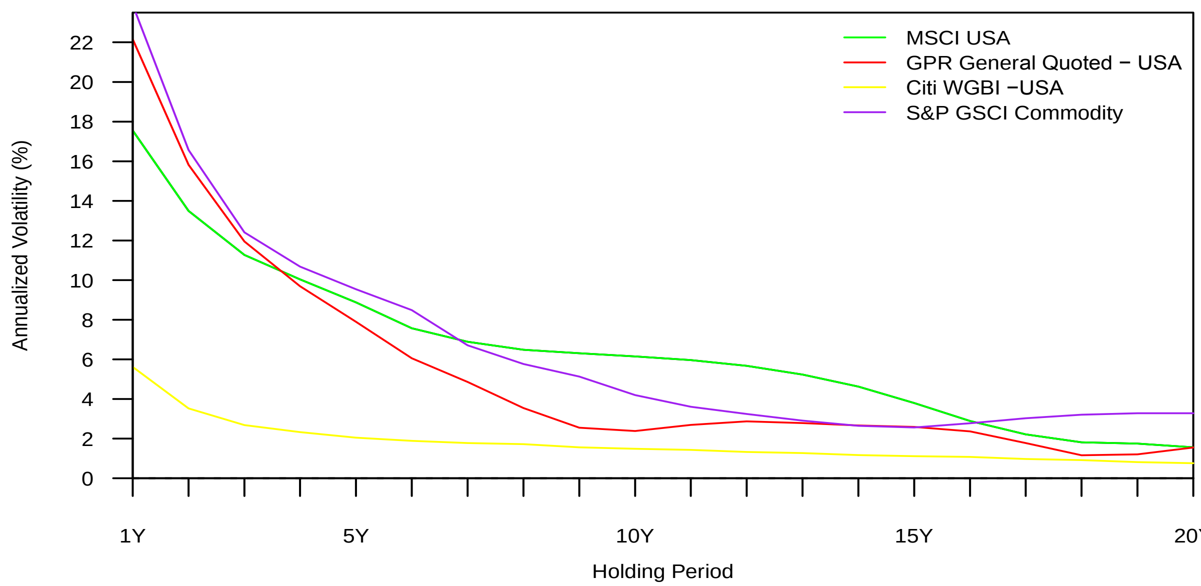
Notes: The correlations are calculated with monthly returns from 1985:1 to 2014:12.

However, as indicated by the literature, correlations appear to vary over time and assuming constant correlations would yield an insufficient picture. Thus, Fig. 10 displays the time-varying correlations, which clearly deviate from the constant mean over several continuous years.
The coefficients between equity and real estate in the first panel decrease from 1990:1 to a minimum of 0.158 but increase after the burst of the dotcom bubble. All indicators finally cross the constant mean during the financial crisis from below and stay more or less constant up to 2013. Intriguingly, the dynamic conditional correlation between real estate and equity moves on a considerably higher level during the transmission period between the dotcom bubble and the financial crisis. This might be due to the better recognition of the higher volatility of both assets through the GARCH model. Nevertheless, all indicators show that the diversification potential of real estate in combination with equity decreased during the financial crisis. However, the most recent development shows that the DCC indicator decreased and crossed the constant correlation from above. The other conditional indicators also slightly decreased, which might be a sign for improving diversification possibilities and the possibility of mean reversion in correlation coefficients over longer time periods.
The correlation with government bonds is in general very low and mainly stays within the range of -0.25 and +0.25. Negative correlations can be observed over long time periods with the desirable tendency to decrease during times of turmoil. This phenomenon is similar for the correlation between stocks and bonds and has been confirmed by several studies. However, the constant correlation is slightly higher compared to the constant correlation between equity and bonds, which might be seen as a disadvantage of real estate when competing against equity.
Finally, the correlation between real estate and commodities depict the highest variation, ranging from -0.6 to 0.5. Panel three further depicts strongly decreasing diversification benefits during times of high volatile markets, as all indicators soared around 0.5 during the financial crisis.

Overall, the correlation analysis shows promising diversification benefits through adding listed real estate to a mixed-asset portfolio. Even after the increase in correlations with equity and commodities the coefficients seem to move back to the low constant correlations. Nevertheless, a portfolio manager should regularly revaluate the diversification benefits of the asset class and consequently adjust the portfolio weights. Portfolio managers especially have to be aware of the reaction to volatile markets and tail dependencies.

When speaking about potential diversification benefits time diversification has to be analyzed as well. The main idea of time diversification is that the risk decreases with the holding period and the term structure of risk decreases (Campbell & Viceira, 2005). Hence, Fig. 9 displays the term structure of risk for the national indices. All four indices show decreasing annualized volatility (SD) with increasing investor horizon. However, the real estate investment inhibits the strongest decrease. In the beginning, real estate and commodity have the highest volatility. However, a horizon of five years already leads to a lower volatility for real estate compared to equity and commodities.

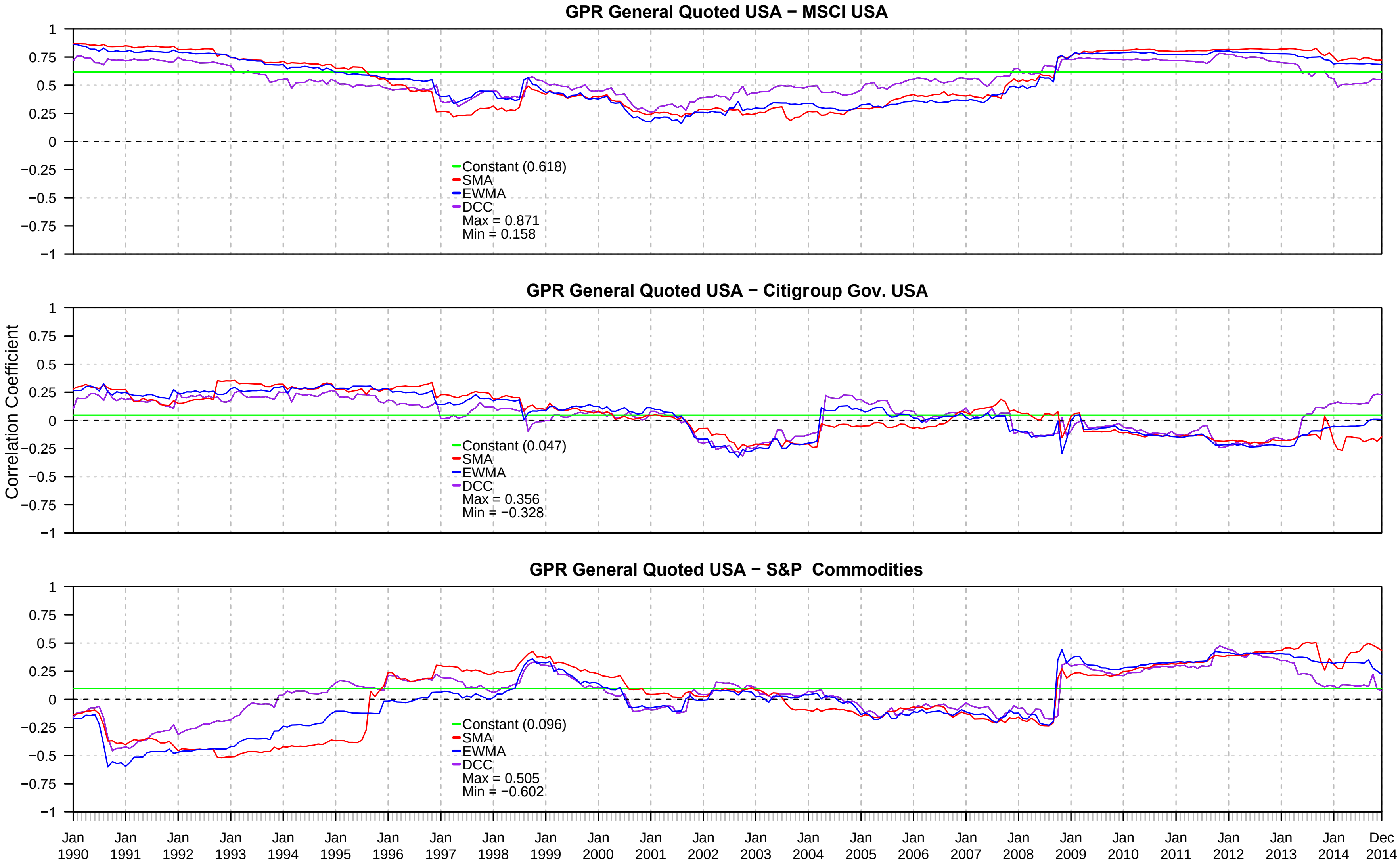
Fig. 9:
National – Term Structure of Risk



Notes: The annualized volatility calculations are based on monthly returns. Furthermore, all starting points for the holding periods within the sample period 1985:1-2014:12 are considered. Nevertheless, the longer the holding period gets, the more difficult is a reliable calculation due to limited observations.

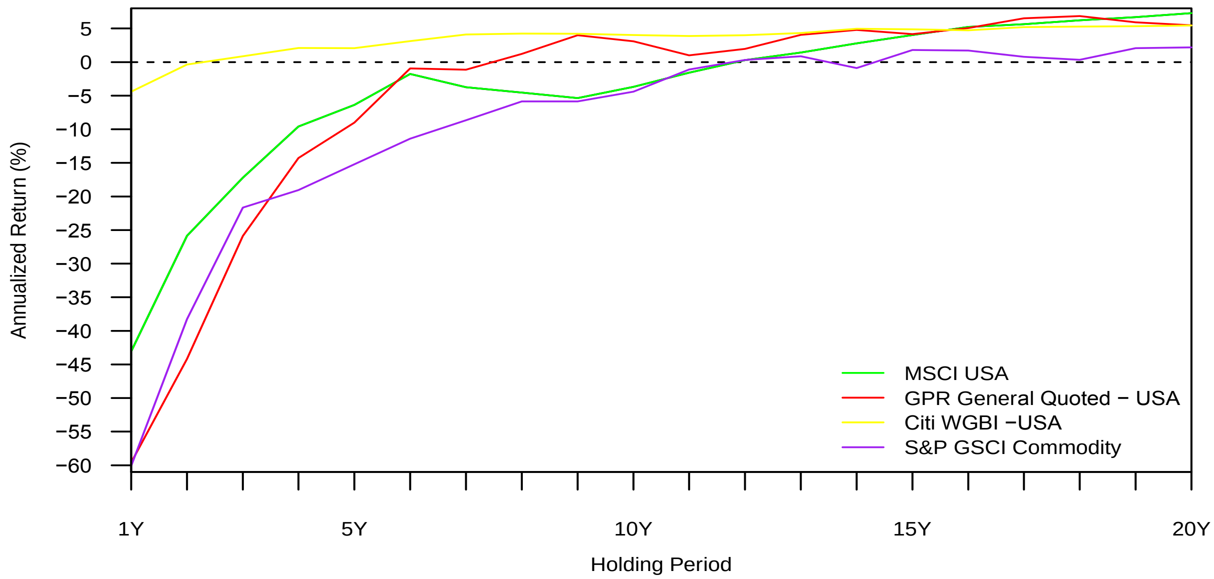
One major disadvantage of Fig. 9 is that it disguises the maximum downside risk. Thus, Fig. 11, based on the ideas of Bennyhoff (2008), shows the minimum annualized return conditional on the holding period. For the whole sample the notion of time diversification becomes ambiguous. On the one hand, the risk of the investments is decreasing with the holding period. On the other hand, neither equity nor real estate do clearly outperform the much more stable government bond investment in the worst-case scenarios with holding periods up to 15 years. Nevertheless, the graph shows that real estate is earlier able to guarantee a positive annualized return compared to commodities and equities. This advantage vanishes when looking at longer holding periods.

Fig. 10:
National – Correlations with Real Estate over Time



Notes: The calculations are based on monthly returns from 1985:1 to 2014:12. For the SMA a window of 15 years is used. Moreover, the decay factor for the EWMA was set to 0.97.

Fig. 11:
National – Minimum annualized Returns depending on Holding Periods



Notes: The figure shows the minimum annualized return depending on the holding period. Furthermore, all starting points for the holding periods within the sample period 1985:1-2014:12 are considered.

Tests for stationarity, as proposed by Stevenson (2002) and Balvers *et al.* (2000), weaken the idea of mean reversion. Table 6 presents the results of the conducted tests. Neither the ADF test nor the PP test can reject the null hypothesis of a unit root for one of the indices.

Table 6:
National – ADF and PP Tests

	Equity	**Real Estate**	**Gov. Bonds**	**Commodities**
Teststatistic	1.27	0.93	0.66	-2.16
Crit(1%)	-3.44	-3.44	-3.44	-3.44
Crit(5%)	-2.87	-2.87	-2.87	-2.87
Crit(10%)	-2.57	-2.57	-2.57	-2.57
PP Test (P-Value)	0.94	0.81	0.42	0.28

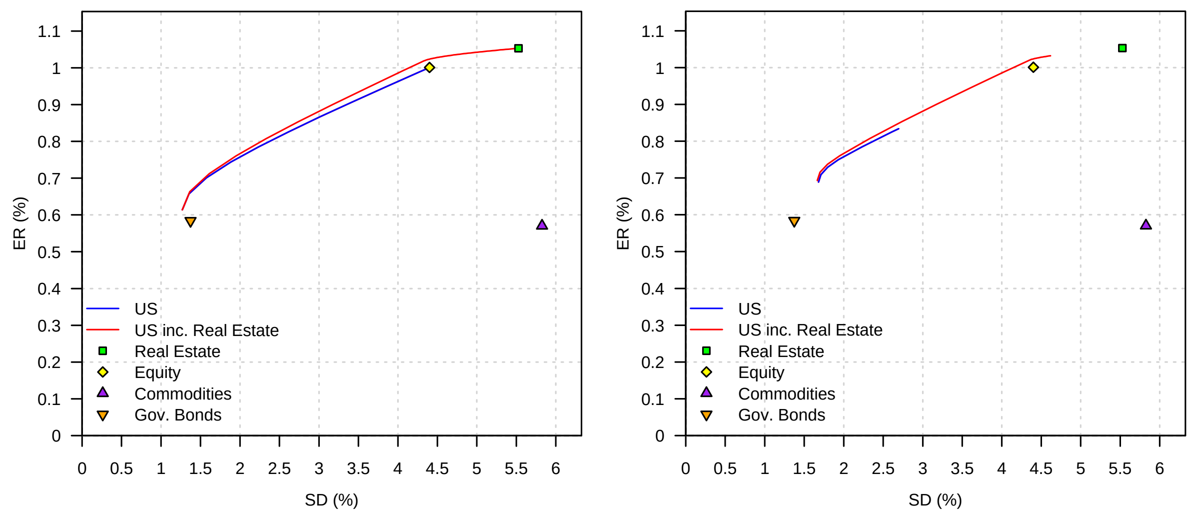
Notes: Scaled level data of the respective indices from 1985:1 to 2014:12 is used for the tests. The AIC selection criterion determines the number of lags for the ADF tests. For the ADF tests a drift component is added. Furthermore, tests with a trend component have also been conducted but do not lead to different outcomes.

Overall, the results do not yield strong evidence for time diversification in terms of tests for stationarity. However, the plotting exercises indicate benefits for all assets. In particular, real estate seems to be of advantage for mid-term investors with horizons between five and fifteen years. This observation is strengthened through very low constant correlations for mid-term holding periods (five to fifteen years), indicating strong diversification benefits of a medium term real estate investment within a mixed portfolio (for details see Appendix F).

5.1.3 Portfolio Optimization

The following portfolio optimization exercises can be seen as a test of parts of the individual analyses from above bringing return enhancing and diversification together. Fig. 12 shows the efficient frontiers for portfolios with and without national real estate. At first glance there is not a significant benefit from adding real estate considering the whole sample period.

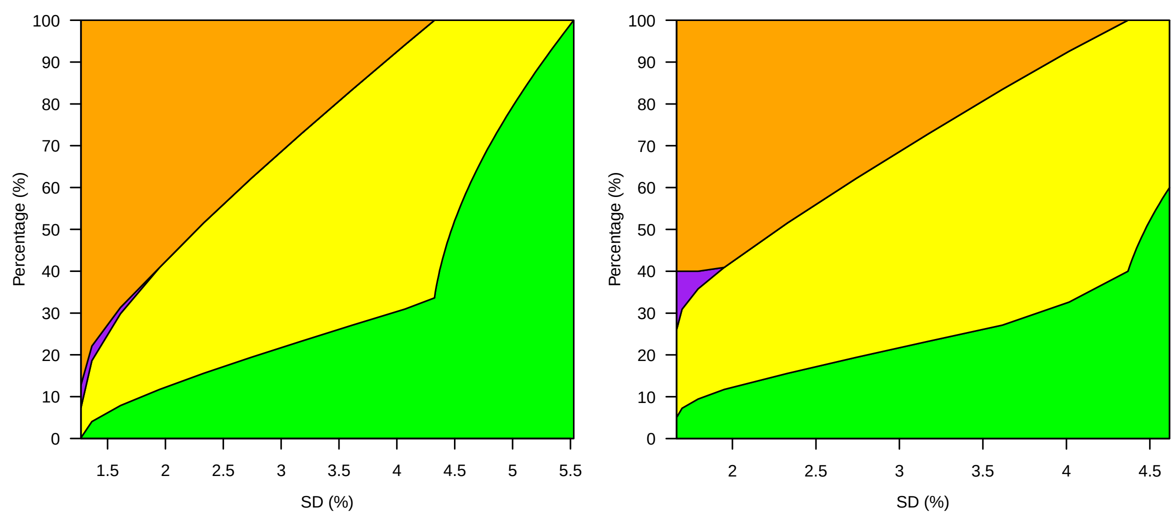
Fig. 12:
National – Efficient Frontiers



Notes: The graphs show the mean-variance efficient frontiers for two portfolios, which consist exclusively of long and passive US investments. The frontiers on the right hand side are further constraint with a maximum allocation of sixty percent per asset class. Monthly returns from 1985:1 to 2014:12 are used for the optimization.

Nevertheless, the change in weights along the frontier, displayed in Fig. 13, promotes an inclusion of real estate. If controlling for a maximum allocation of 60 percent to one asset, mean-variance optimization calls for an investment along the whole frontier.

Fig. 13:
National – Optimal Weights



Notes: The figures display the weight allocation corresponding to Fig. 12 dependent on SD.

The allocation to real estate rises from around five percent in the minimum variance portfolio to over 50 percent in high-risk portfolios. Hence, portfolio managers, who want to use real estate as an absolute return enhancer and are willing to take a higher risk on the way, may benefit from an investment. Furthermore, investors might be able to further diversify their portfolios.

However, previous ideas are challenged by the mean-variance spanning tests in Table 7. The results do not show a statistically significant benefit from adding real estate. Even the higher risk portfolios are not significantly different.

Table 7:
National – Spanning Tests

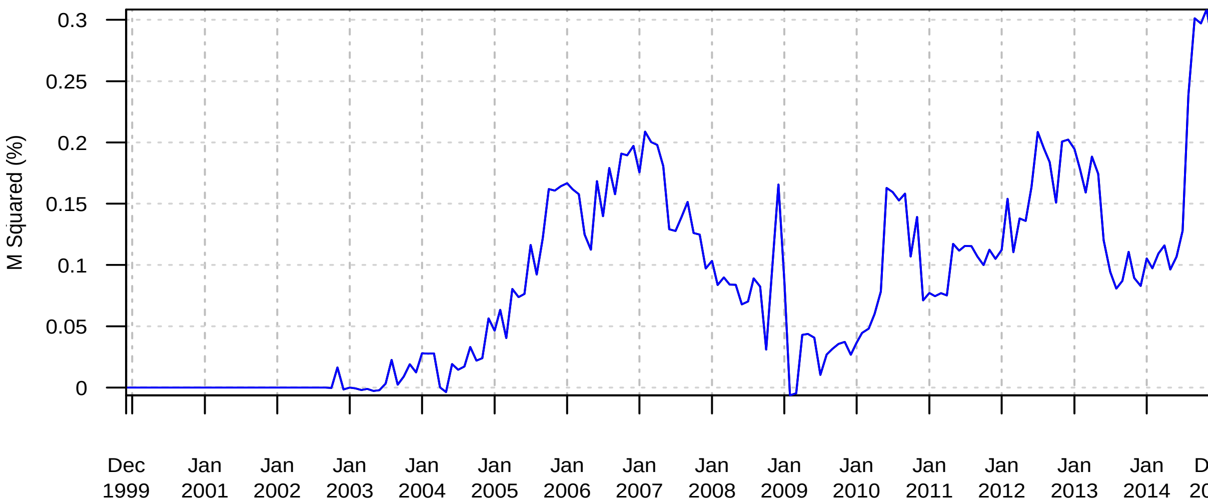
Portfolio	**Intercept**			**Coefficient Benchmark Porfolio**		
	Value	t-stat	Prob.	Value	t-stat	Prob.
MVP	0.000	0.559	0.576	1.000	-0.098	0.922
SD2	0.000	0.866	0.387	0.961	-1.194	0.233
SD2.5	0.000	0.821	0.412	0.956	-1.126	0.261
SD3	0.001	0.811	0.418	0.950	-1.167	0.244
SD3.5	0.001	0.828	0.408	0.944	-1.291	0.198
SD4	0.001	0.797	0.426	0.944	-1.263	0.207
60 % per Asset Constraint						
	Value	t-stat	Prob.	Value	t-stat	Prob.
MVP	0.000	0.931	0.353	0.982	-1.245	0.214
SD2	0.000	0.866	0.387	0.961	-1.194	0.233

Notes: The spanning tests are conducted with the monthly returns of optimized portfolios. Newey-West standard errors are considered and return data from 1985:1 to 2014:12 is used.

In order to test if there are changes in the benefits of adding real estate to a portfolio, further rolling mean-variance optimization with a fifteen-year window are conducted. For that, a portfolio manager with a target SD of two percent is assumed.[38] This approach is based on the ideas of Kroencke and Schindler (2012) and enables investors to get an impression about the time-varying benefits. In contrast to Kroencke and Schindler (2012), the performance measure M Squared is plotted against time. The risk-free rate for deleveraging purposes is represented by the prevailing one-month Treasury bill rate. Fig. 14 shows that a portfolio manager, who included real estate, was able to form portfolios with higher expected return-risk ratios during several time frames. However, the benefits were almost zero up to the beginning of 2004. Obviously, the burst of the dotcom bubble made a real estate investment much more attractive and led to an increase of around 20 basis points. The financial crisis again changed the picture and M Squared temporarily plummeted to zero. Nevertheless, the most recent development should be of high interest, as M Squared peaked at the end of 2014 with around 30 basis points.

[38] As a mean-variance mixed-asset (bonds and equity) investor with an average risk-aversion level would invest in a portfolio with an expected SD of 1.864 percent per month. The choice seems to be feasible.

Fig. 14:
National – Time-varying M Squared



Notes: The figure depicts M Squared comparing portfolios with and without a real estate investment. A portfolio manager with a target SD of two percent is assumed. Expected return and covariance matrix are calculated within a 15 years fixed rolling window. Monthly return data from 1985:1 to 2014:12 is used.

The previous empirical observations built on in-sample estimations, which do not consider that a portfolio manager can only use all ex post available information for choosing the right asset allocation. Thus, out-of-sample tests, like in Kroencke and Schindler (2012), are conducted. Table 8 presents the results for monthly-rebalanced portfolios. The results are in line with the observation that real estate can only enhance high-risk portfolios on the upper end of the frontier. However, the highest realized mean return and return-risk ratios can be identified for the lower risk portfolios questioning a real economical benefit of adding real estate. In particular, the realized SD of the extended portfolios is significantly higher than the targeted SD highlighting the problematic of high tail risk and volatility of a real estate investment.

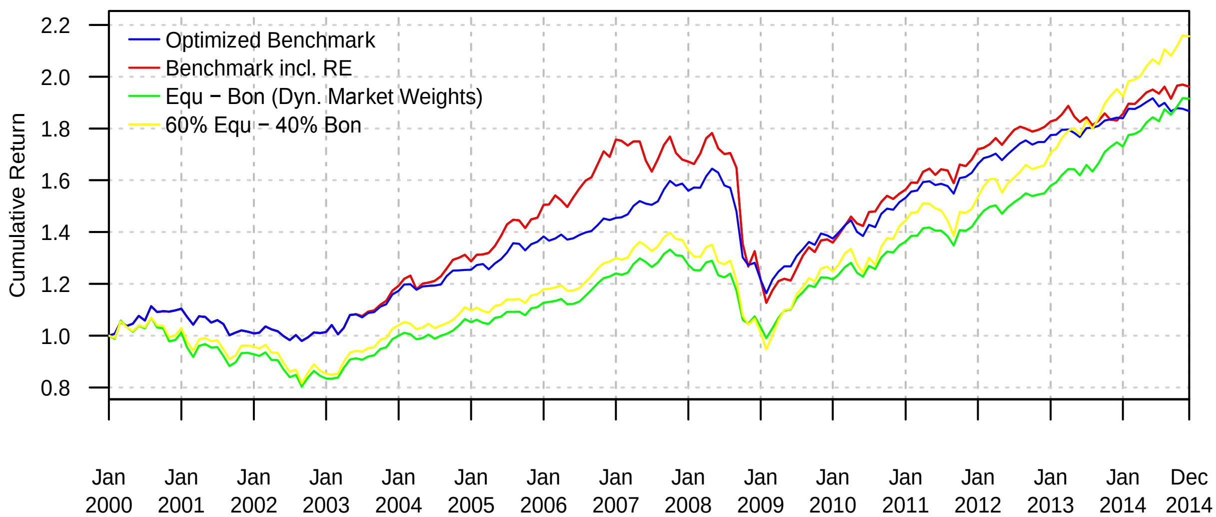
Table 8:
National – Out-of-Sample Tests

		Without Maximum Allocation			**Maximum Allocation 60% per Asset**		
Target SD (%)	Portfolio	Mean Return	SD (%)	Ratio	Mean Return (%)	SD (%)	Ratio
1.5	Benchmar	0.376	1.368	0.275			
	incl. RE	0.415	1.768	0.235			
2	Benchmar	0.297	1.885	0.158	0.363	1.991	0.182
	incl. RE	0.389	2.535	0.153	0.405	2.587	0.156
2.5	Benchmar	0.220	2.396	0.092	0.288	2.483	0.116
	incl. RE	0.351	3.259	0.108	0.361	3.290	0.110
3	Benchmar	0.166	2.862	0.058	0.294	2.996	0.098
	incl. RE	0.332	3.908	0.085	0.396	3.944	0.100
3.5	Benchmar	0.109	3.247	0.033	0.247	3.274	0.075
	incl. RE	0.255	4.475	0.057	0.363	4.216	0.086
4	Benchmar	0.053	3.358	0.016	0.247	3.274	0.075
	incl. RE	0.198	4.687	0.042	0.366	4.257	0.086

Notes: The table shows the descriptive statistics of monthly-rebalanced portfolios with different target SDs. The weighting of assets is based on mean-variance optimization within a 15 years rolling window. The sample period spans 1985:1-2014:12.

However, to get a better understanding in which periods the out-of-sample strategy was successful, Fig. 15 shows the cumulative returns of the two portfolios compared to two benchmark portfolios. The development substantiates earlier results and reflects a similar pattern as Fig. 14. The chart also depicts that the optimization procedure successfully outperformed the two benchmarks during several periods. On the other hand, the weaknesses of mean-variance optimization, with the assumption of normally distributed returns and the difficulty to predict tail risk, are revealed. The optimized portfolio with real estate inhibits a higher ES (around four percent higher) and a higher maximum drawdown (around six percent higher).

Fig. 15:
National – Cumulative Returns of optimized Portfolios (Fixed Window)

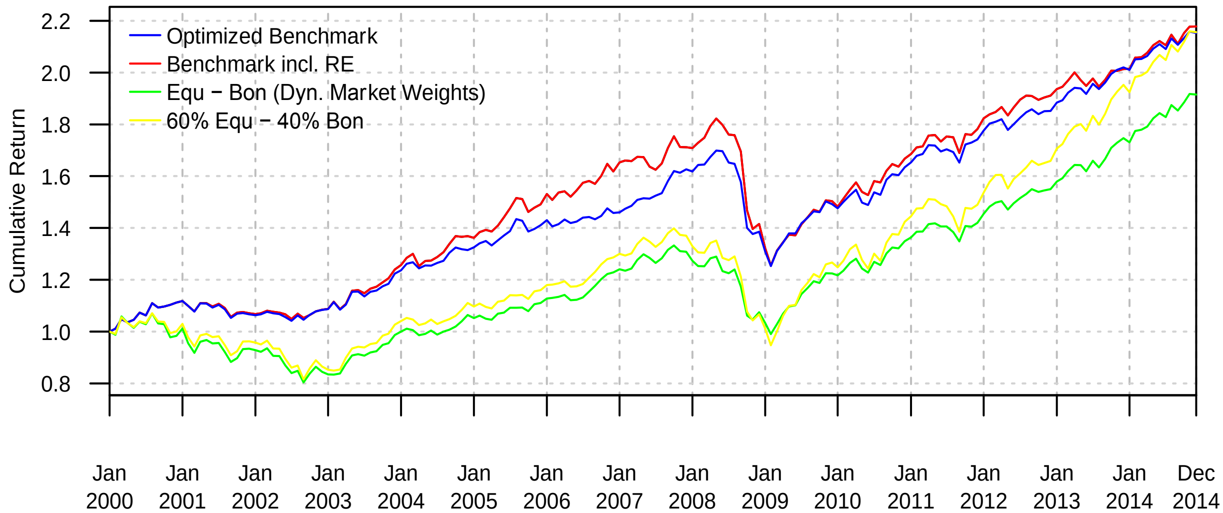


Notes: The assumed investor targets a SD of two percent and rebalances his portfolio every month based on the information within a 15 years fixed window. The allocation is constraint to 60 percent per asset class and short selling is not allowed. Monthly returns from 1985:1 to 2014:12 are used.

Fig. 16 shows the results of a different optimization approach. In this case the portfolio manager uses all past observations and not only the observations within the fixed window to optimize the portfolio. Basically, this approach simulates a portfolio manager, who has additional information every month but does not forget previous developments. In general, the strategy seems to be more promising as the performance gap compared to the two benchmarks widens. However, when comparing the two optimized portfolios in terms of return-risk ratio and ES (around four percent difference), it becomes clear that the portfolio without listed real estate performed better considering the whole sample period.[39]

[39] The corresponding time-varying weights can be found in Appendix B.

Fig. 16:
National – Cumulative Returns of optimized Portfolios (Expanding Window)



Notes: The portfolio manager targets a SD of two percent and rebalances his portfolio every month based on the information within the gradually increasing window (initially 15 years). Furthermore, the allocation is constraint to 60 percent per asset class and short selling is prohibited. Monthly return data from 1985:1 to 2014:12 is used.

On the one hand, the portfolio optimization and the out-of-sample optimization do not suggest significant benefits of adding real estate considering the whole sample period. On the other hand, the time-varying analysis highlights benefits for several subperiods. However, that would imply that investors are capable of timing the market accordingly.

5.1.4 Inflation Hedging

In a first step, the tests for short-term hedging capabilities of the four asset classes are discussed. Then, the long-term hedging capabilities are evaluated by means of the introduced error correction model. Intuitively, investors would expect a close relation of inflation with stock prices of real estate companies, as real assets in form of properties back their value. In the short run this intuition does not hold based on the following empirical results.

The regression results for the contemporaneous relation between the returns of the national indices and the total realized inflation, expected, and unexpected inflation are displayed in Table 9 and Table 10. The insignificant coefficients of realized inflation for the regression with real estate are negative as well as positive for the three time frames. However, when dividing total inflation into expected and unexpected inflation the results are similar to previous studies (e.g. Hoevenaars *et al.*, 2008; Froot, 1995). The coefficients of expected inflation for real estate and equity are negative but not significant for all subsamples. Furthermore, both tables suggest that a commodity investment has strong hedging capabilities for total inflation risk, as all coefficients are clearly positive and significant at a one percent level. This observation was expected as both the CIP and the S&P GSCI consist of similar constituents. Moreover, the S&P GSCI is recognized as a measure for price movements. Intriguingly, a bond investment is not able to protect against inflation in the short run. The negative and significant coefficient of total inflation rather implies perverse hedging capabilities. Table 10 further points out that only unexpected inflation is significantly correlated with the total returns of the bond indices. A possible explanation for the negative correlation might be a dominating price effect within the total return of the bond index. The high duration and therefore high interest sensitivity of the used bond index undermines this idea.

Table 9:
National – Contemporaneous Fisher Regressions

		Sample: 1985:1 -2014:12			Sample: 1985:1 - 1999:12			Sample: 2000:1 - 2014:12		
Dep. Var.	**Exp. Var.**	**Coeff.**	**Prob.**		**Coeff.**	**Prob.**		**Coeff.**	**Prob.**	
Real Estate	Constant	0.033	0.105		0.076	0.020		0.028	0.263	
	Inflation	-0.036	0.987		-6.083	0.102		1.671	0.440	
Equity	Constant	0.027	0.094	*	0.077	0.001	**	0.007	0.684	
	Inflation	0.525	0.750		-3.676	0.117		1.131	0.504	
Bonds	Constant	0.024	0.000	***	0.035	0.000	**	0.019	0.000	**
	Inflation	-0.985	0.001	***	-1.647	0.028	**	-0.972	0.006	***
Commodities	Constant	-0.060	0.000	***	-0.085	0.024	***	-0.051	0.001	**
	Inflation	11.873	0.000	***	14.177	0.001	***	11.481	0.000	***

Notes: The table shows the results of the Fisher regression (equation twenty) based on quarterly returns. HAC robust standard errors are used to calculate the probabilities. The significance of the coefficients at different confidence level is indicated by the number of asterisks: * 10%, ** 5%, and *** 1%.

Additionally, rolling regressions over a fifteen-year window are conducted in order to control for different regimes. Fig. 17 plots the coefficient of expected inflation over time. The color of the dot indicates if the coefficient is statistical significant. The upper panel shows the results of the Fama-Schwert equation. As expected, most of the coefficients are negative and only two observations are statistically significant at a ten percent level. In the lower panel economic control variables have been added to consider a potential proxy mechanism. Apparently, the curve shifts upwards and much more positive coefficients can be identified. Furthermore, significant coefficients for expected inflation can be observed between 2003 and 2007. However, the majority stays insignificant. Hence, despite including control variables robust evidence for short-term hedging capabilities of real estate cannot be identified for the whole sample period.

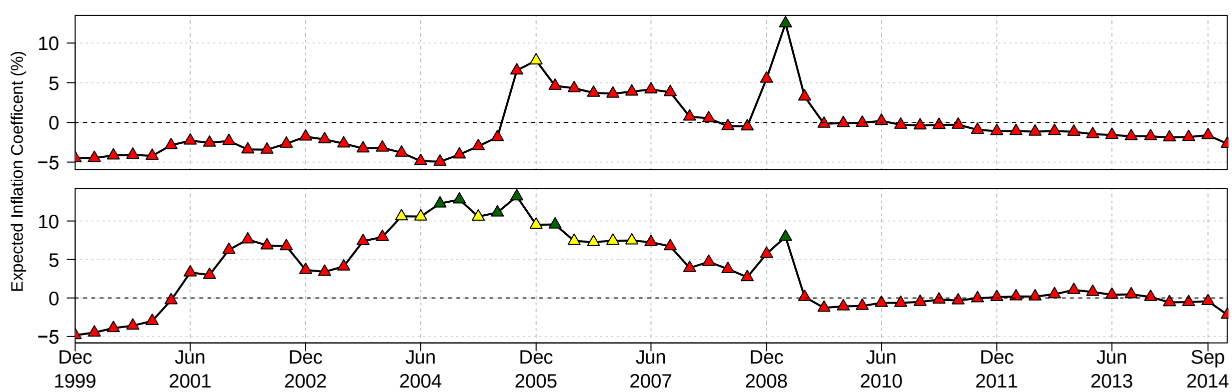
Table 10:
National – Fama-Schwert Regressions

Dep. Var.	Exp. Var.	Sample: 1985:1 - 2014:12 Coeff.	Prob.		Sample: 1985:1 - 1999:12 Coeff.	Prob.		Sample: 2000:1 - 2014:12 Coeff.	Prob.	
Real Estate	Constant	0.055	0.105		0.061	0.143		0.055	0.334	
	Expected Inflation	-3.436	0.419		-4.491	0.404		-3.386	0.659	
	Unexpected Inflation	0.777	0.708		-7.977	0.068	*	2.524	0.107	
Equity	Constant	0.041	0.035	**	0.071	0.001	**	0.036	0.237	
	Expected Inflation	-1.593	0.482		-2.958	0.270		-3.648	0.360	
	Unexpected Inflation	1.025	0.544		-4.586	0.231		2.007	0.130	
Bonds	Constant	0.013	0.004	***	0.022	0.032	**	0.011	0.026	**
	Expected Inflation	0.555	0.311		-0.122	0.905		0.293	0.702	
	Unexpected Inflation	-1.441	0.000	***	-2.722	0.001	***	-1.270	0.000	***
Commodities	Constant	-0.084	0.000	***	-0.084	0.003	***	-0.052	0.029	**
	Expected Inflation	14.319	0.000	***	14.319	0.000	***	11.202	0.001	***
	Unexpected Inflation	14.595	0.000	***	14.595	0.010	**	11.391	0.000	***

Notes: The table shows the results of the Fama-Schwert regression based on quarterly returns. Expected inflation is calculated by using the model of Fama and Gibbons (1984). Furthermore, Newey-West standard errors are used. Significance of the coefficients at different confidence level is indicated by the number of asterisks: * 10%, ** 5%, and *** 1%.

When looking at realized inflation and unexpected inflation the picture gets more disillusioning. Neither rolling realized inflation nor unexpected inflation coefficients are positive when controlling for economic variables (see Appendix C). It seems like there is a dominant price effect, which drives down returns due to unanticipated inflation. Another possible explanation might be that unanticipated inflation comes with unexpected increasing costs. Hence, a slight decrease in stock prices would be the consequence and might explain the perverse relationship.

Fig. 17:
National – Coefficients of expected Inflation from rolling Regressions



Notes: The graphs depict the coefficient of the expected inflation rate and the real estate return as the dependent variable. The upper graph is based on the Fama-Schwert regression (equation 24). The graph below shows the results after controlling for macroeconomic factors. The regressions are based on a fixed 15 years window of quarterly returns. Green stands for significant at the 5 percent level, yellow at the 10 percent level, and red for below the 10 percent level.

For many investors the disappointing observations for the short-run may not be as important as the following long-term empirical insights. Table 11 displays the results of the level regressions, as part of the error correction model, for all four-asset classes. Despite an insignificant coefficient of expected inflation for equity, real estate has a positive and also statistically significant coefficient at a five percent level. Therefore, it seems like real estate can offer long-term investors better inflation hedging capabilities compared to equity. As expected, bonds are also able to hedge against expected inflation in the long run. The coefficient is even significant at a one percent level but much lower compared to the coefficient of real estate. Intriguingly, the coefficient of expected inflation for commodities is insignificant, which should be due to the drop in prices and the absent recovery of commodities in recent years.[40] This idea is further substantiated by the fact that for level regressions, which exclude years above 2006, the positive coefficients for both expected and unexpected inflation become highly significant. For all four regressions stationary error terms are identified undermining an equilibrium relationship between the dependent and significant explanatory variables.
The first difference regressions are displayed in Table 12. After controlling for economic variables, the coefficient for expected inflation for real estate, equity and commodities is clearly positive. This diminishes the idea of a perverse inflation hedge. However, the coefficients for real estate and equity are not significant denying the short-term hedging capabilities. This and also the results for bonds and commodities are in line with the results from above. As expected, the ECT is negative and significant for all four regressions indicating a return to the identified equilibrium relationships over time. Hoesli *et al.* (2008) have similar results for the first difference regressions for REITs in the USA and real estate companies in the UK.[41] In contrast to their results, the ECT is higher for real estate predicting slightly better hedging capabilities and a faster reversion to equilibrium. However, the value of the ECM in all regressions is below 0.4 suggesting a rather slow adjustment back to equilibrium. Thus, even mid-term investors should be careful with a too enthusiastic interpretation and be aware of longer deviations from equilibrium.[42]

In conclusion, the inflation analysis emphasizes the poor short-term hedging capabilities of real estate. Nevertheless, real estate seems to be in a long-term relationship with expected inflation suggesting inflation hedging capabilities with increasing investment horizon. Moreover, the results suggest no hedging capabilities of equity in the long run. This would further boost the allocation towards real estate if directly competing against equity.

[40] Using realized inflation, instead of splitting the rate, yields similar results.
[41] Hoesli *et al.* (2008) sample goes only up to 2003.
[42] A brief discussion about the order of integration of the used variables and potential problems can be found in Appendix D.

Table 11:
National – Error Correction Model – Level Regressions

	Real Estate			Equity			Bonds			Commodities		
	Sample: 1985:1 - 2014:12			Sample: 1985:1 - 2014:12			Sample: 1985:1 - 2014:12			Sample: 1985:1 - 2014:12		
	Coeff.	**Prob.**		**Coeff.**	**Prob.**		**Coeff.**	**Prob.**		**Coeff.**	**Prob.**	
Constant	0.632	0.007	***	-0.259	0.044	**	0.257	0.000	***	0.136	0.589	
Expected Inflation	5.793	0.026	**	-0.130	0.911		1.304	0.000	***	3.832	0.207	
Unexpected Inflation	1.196	0.731		-4.127	0.032	**	-0.379	0.528		16.605	0.000	***
MSCI World	0.439	0.037	**	0.840	0.000	***	-0.288	0.000	***	0.357	0.380	
MSCI USA	-0.289	0.355					0.208	0.000	***	-0.739	0.116	
Industrial Production USA	-0.244	0.792		2.246	0.000	***	-0.295	0.228		0.997	0.594	
M2	-1.269	0.032	**	0.173	0.587		0.105	0.530		-1.398	0.219	
Real GDP USA	5.938	0.001	***	-2.564	0.089	*	0.787	0.160		1.082	0.816	
Libor	-7.685	0.000	***	-0.213	0.959		0.111	0.860		0.927	0.858	
Treasury Bill 3 Month	0.127	0.937		1.318	0.620		0.277	0.519		0.061	0.987	
Adjusted R squared:	0.978			0.989			0.910			0.910		

Notes: The table presents the results of the level regressions (equation 26). For the regressions quarterly logged and scaled values are used. All indices are scaled to one in 1985:1 to avoid any scaling effects. Furthermore, logarithms are used to make first differences, as continuous returns, better interpretable. For calculating the confidence levels of the displayed coefficients Newey-West standard errors are used. The level of significance of the coefficients is indicated by the number of asterisks: * 10%, ** 5%, and *** 1%.

Table 12:
National – Error Correction Model – First Difference Regressions

	Real Estate			Equity			Bonds			Commodities		
	Sample: 1985:1 - 2014:12			Sample: 1985:1 - 2014:12			Sample: 1985:1 - 2014:12			Sample: 1985:1 - 2014:12		
	Coeff.	**Prob.**		**Coeff.**	**Prob.**		**Coeff.**	**Prob.**		**Coeff.**	**Prob.**	
Constant	0.032	0.156		0.004	0.662		0.006	0.386		-0.063	0.000	***
Expected Inflation	1.446	0.559		0.495	0.639		-0.280	0.747		15.650	0.000	***
Unexpected Inflation	-1.033	0.219		-1.022	0.001	**	-1.021	0.002	***	12.177	0.000	***
MSCI World	0.286	0.015	**	0.911	0.000	***	-0.017	0.821		0.556	0.006	***
MSCI USA	0.408	0.011	**				-0.065	0.407		-0.457	0.004	***
Industrial Production USA	-0.414	0.516		0.065	0.800		-0.114	0.458		1.313	0.022	**
M2	-2.236	0.014	**	-0.192	0.585		0.169	0.517		-0.796	0.313	
Real GDP USA	2.564	0.087		0.254	0.617		0.921	0.011	**	-3.055	0.008	***
Libor	-10.089	0.154		-4.548	0.154		0.811	0.642		-10.124	0.034	**
Treasury Bill 3 Month	3.209	0.463		2.409	0.231		0.113	0.925		5.295	0.107	
ECT	-0.282	0.000	***	-0.154	0.011	**	-0.342	0.000	***	-0.201	0.001	***
Return (t-1)	0.257	0.000	***	0.081	0.048	**	0.087	0.327		0.012	0.881	
Return (t-2)	-0.013	0.798		-0.024	0.427		-0.024	0.758		-0.012	0.823	
Adj. R squared	0.560			0.851			0.335			0.667		

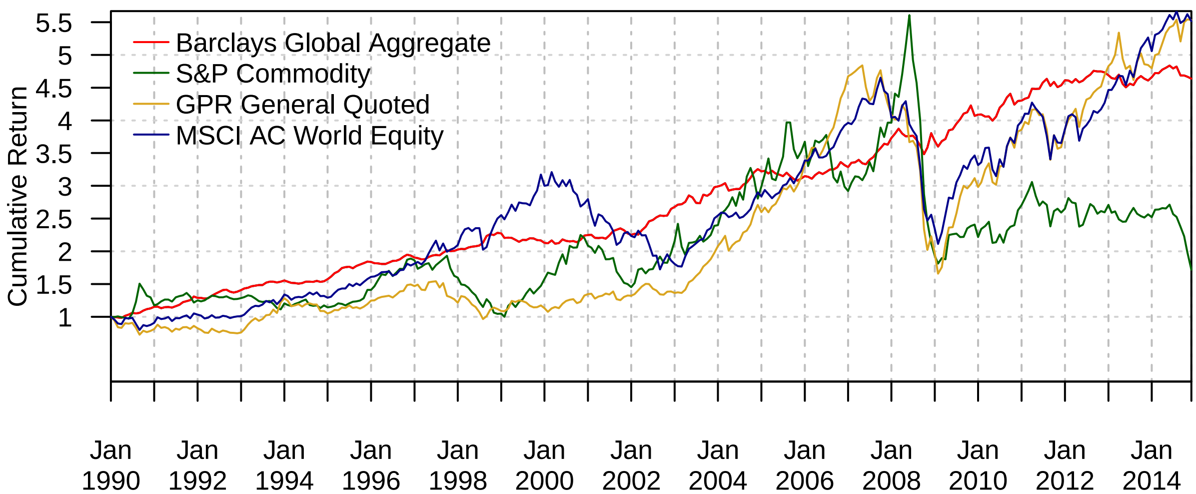
Notes: The table depicts the first difference regression (equation 27) results. The lagged residuals of the level regressions are used as the ECT. Additionally, lagged values of the continuous returns of the respective index are included to control for potential autocorrelation. For calculating the confidence levels of the coefficients Newey-West standard errors are used. The significance of the coefficients is indicated by the number of asterisks: * 10%, ** 5%, and *** 1%.

5.2 Global Portfolio

5.2.1 Return Enhancement

Like for the national analysis, the study gives an overview about the performance and risk of the individual global indices. Fig. 18 depicts the cumulative returns of the four different asset classes. The picture changed mainly because of the more moderate performance of the global real estate and equity indices over the time period 1990:1 to 2014:12. This leads to a relative better performance of the global bond index and questions an international extension of a national portfolio at first sight.

Fig. 18:
Global – Cumulative Returns of global Indices



Notes: Monthly returns from 1990:1 to 2014:12 are used to calculate the respective cumulative returns.

Table 13 confirms this observation with global real estate, equity and bond indices having lower mean returns and lower return-risk ratios compared to the US indices. Thus, a broad global index investing strategy does not seem to be of advantage compared to the national opportunity set. In particular, risk-adjusted return enhancing capabilities of a global real estate investment cannot be identified. Nevertheless, when solely comparing the global property indices with the global equity indices, similar conclusions as on the national level can be drawn. The property indices have slightly higher mean returns but also higher SDs. Besides the GPR 250 REIT index, the indices display smaller return-risk ratios, which might be due to the high share of US stocks in the GPR 250 REIT index. It becomes clear that it is difficult to make an explicit recommendation for the return enhancing capabilities of the global real estate indices when having national indices as an alternative.

Table 13:
Global – Descriptive Statistics of monthly Returns

	Real Estate			Equity			Bonds		
	Mean	**SD**	**Ratio**	**Mean**	**SD**	**Ratio**	**Mean**	**SD**	**Ratio**
National									
USA	0.996	5.691	0.175	0.871	4.244	0.205	0.511	1.280	0.399
International									
Australia	1.010	5.648	0.179	0.935	5.962	0.157	0.754	3.403	0.222
Canada	0.067	6.801	0.010	0.848	5.589	0.152	0.625	2.563	0.244
France	0.921	5.802	0.159	0.735	5.865	0.125	0.642	3.052	0.210
Germany	0.419	7.489	0.056	0.808	6.591	0.123	0.573	3.037	0.189
Hong Kong	1.434	10.062	0.142	1.182	7.362	0.161			
Japan	0.467	8.463	0.055	0.053	5.724	0.009	0.431	3.368	0.128
Netherlands	0.629	5.666	0.111	0.883	5.623	0.157	0.598	3.047	0.196
Singapore	1.235	10.310	0.120	0.840	7.156	0.117	0.451	1.992	0.226
Sweden	0.896	9.722	0.092	1.114	7.526	0.148	0.556	3.394	0.164
UK	0.744	6.264	0.119	0.715	4.744	0.151	0.685	2.898	0.236
Barclays Global Aggregate							0.520	1.572	0.331
GPR General Quoted	0.693	5.184	0.134						
GPR 250	0.665	5.223	0.127						
GPR 250 REITs	0.876	4.681	0.187						
MSCI World				0.654	4.376	0.149			
MSCI ACWI				0.656	4.458	0.147			

Notes: For the calculations monthly returns from 1990:1 to 2014:12 are used.

The picture is also substantiated by the regression results in Table 14. The global real estate index does not generate positive and significant alphas. The rolling regression results for alpha presented in Fig. 19 rather suggests that global real estate underperformed and generated negative alphas up to 40 basis points.

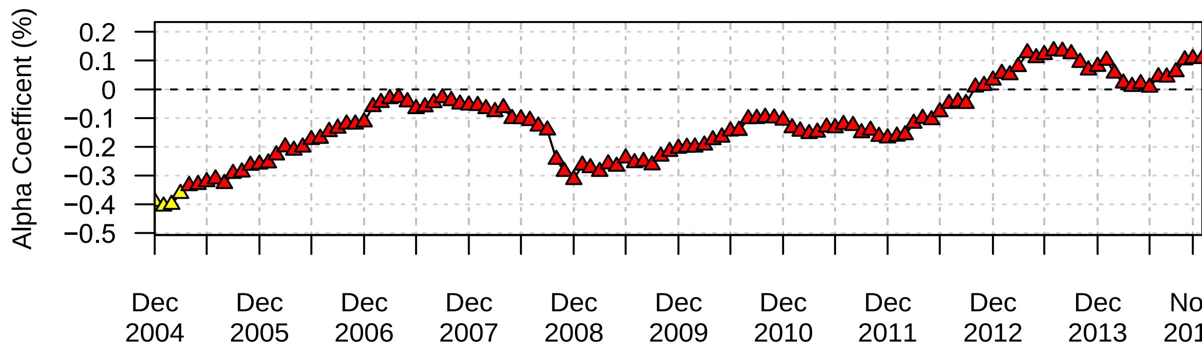
Table 14:
Global – Multi-Factor Model Regressions

	1990:1-2014:12			1990:1-1999:12			2000:1-2014:12		
	Coeff.	Prob.		Coeff.	Prob.		Coeff.	Prob.	
Alpha	-0.002	0.263		-0.006	0.027	**	0.001	0.604	
MSCI ACWI	0.861	0.000	***	1.013	0.000	***	0.802	0.000	***
Barclays Global Aggregate	0.693	0.000	***	0.505	0.009	***	0.769	0.000	***
S&P Commodities	-0.020	0.458		0.057	0.054	*	-0.033	0.394	
SMB	0.259	0.000	***	0.157	0.063	*	0.274	0.000	***
HML	0.437	0.000	***	0.227	0.033	**	0.516	0.000	***
MOM	-0.070	0.060	*	-0.279	0.000	***	-0.047	0.179	
Adj. R squared	0.750			0.730			0.792		

Notes: The table presents the regression results of the multi-factor model (equation 5) with the monthly returns of the GPR General Quoted as the dependent variable. Newey–West Standard Errors are used. The significance of the coefficients at different confidence levels is indicated by the number of asterisks: * 10%, ** 5%, and *** 1%.

These observations are similar to the national outcomes and also speak against adding global real estate for enhancing the risk-adjusted return of a global portfolio. Nevertheless, there also seems to be room for real estate unique risk factors, which can additionally explain the rest of the high volatility of global real estate. However, the coefficients suggest that the return variation is strongly influenced by the global equity index highlighting the close relation between the two asset classes.

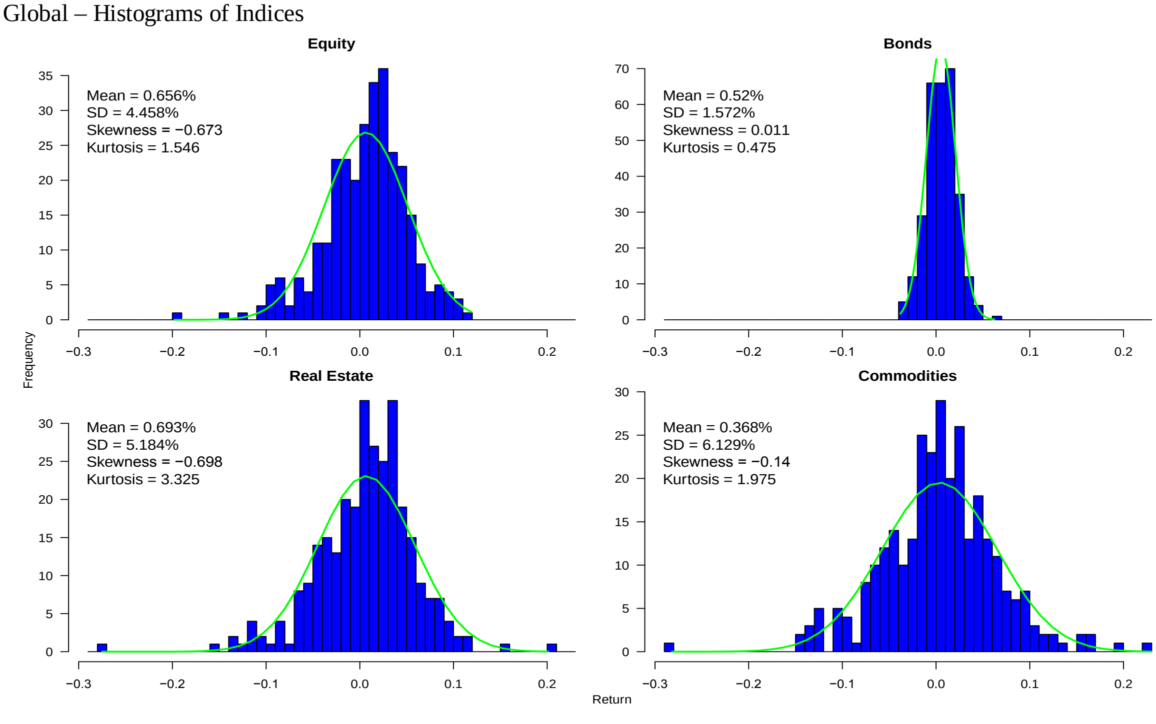
Fig. 19:
Global – Alphas of Real Estate based on rolling Regressions



Notes: The graph depicts the alphas of rolling regressions, following the specifications from Table 14. The regressions are run over a moving window of 15 years. Red dots stand for insignificant alphas, yellow dots for significant at the ten percent level and green dots for significant at the 5 percent level. Monthly returns from 1990:1 to 2014:12 are used.

So far, tail risk was neglected, which might change the first impression of the risk characteristics of the global indices. Fig. 20 shows that the monthly return distribution of the global real estate index has the highest negative skewness and the highest kurtosis.

Fig. 20:
Global – Histograms of Indices



Notes: The graphs depict the histograms of the broad global indices. The monthly returns from 1990:1 to 2014:12 are summed in one percent buckets. In order to make the histograms easier to interpret the normal distribution curve is added in green.

However, all four return distributions come closer to a normal distribution compared to the distributions of the national indices. This already suggests cross-country diversification possibilities. The idea is also confirmed by the ES measures in Table 15. ES for all global real estate indices is significantly lower than for the national real estate index. In contrast, the difference of the values for the global equity indices and the US equity index are rather small. Hence, tail risk would advocate a global real estate index investing strategy.

Table 15:
Global – ES per Month

	Real Estate		Equity		Bonds	
	Cornish Fisher	Empirical	Cornish Fisher	Empirical	Cornish Fisher	Empirical
National						
USA	-19.599	-13.394	-11.534	-10.003	-2.342	-2.350
International						
Australia	-19.433	-14.170	-23.839	-15.460	-9.492	-8.717
Canada	-18.655	-16.500	-15.599	-12.673	-6.003	-5.385
France	-14.162	-12.158	-13.422	-13.624	-5.772	-5.846
Germany	-8.543	-14.253	-15.956	-16.057	-6.108	-6.301
Hong Kong	-14.039	-20.243	-21.195	-16.269		
Japan	-9.469	-16.807	-12.466	-12.044	-5.811	-6.552
Netherlands	-15.306	-12.900	-14.421	-13.453	-6.140	-6.262
Singapore	-13.521	-22.374	-22.027	-17.891	-4.499	-4.288
Sweden	-10.182	-18.202	-16.150	-16.211	-6.479	-6.493
UK	-16.070	-14.092	-11.464	-10.574	-5.637	-6.309
Barclays Global Aggregate					-1.955	-1.984
GPR General Quoted	-13.859	-12.177				
GPR 250	-15.486	-12.591				
GPR 250 REITs	-18.887	-11.758				
MSCI World			-11.004	-10.314		
MSCI ACWI			-10.932	-10.149		

Notes: The table gives an overview of the ES in one month in terms of returns in percent. The calculations are based on the whole sample if possible (1985:1 – 2014:12).

If an investor has access to the individual country indices an improvement in form of risk-adjusted returns stays questionable, as only the Australian real estate index has a better return-risk ratio than the US index. Furthermore, most of the local real estate indices have lower ratios compared to their respective local equity index. Nevertheless, when focusing on the absolute return enhancing capabilities, the three real estate indices of the Asia-Pacific region stand out with high mean returns. However, this possibility would come with high volatility. In general, a global extension of a national portfolio is ambiguous as for equities and bonds no local index can beat the US equivalent with a better return-risk ratio.

Comparing the tail risk of the country indices, it becomes clear that almost all local equity and bond indices have higher ESs compared to their US counterpart. This observation is very interesting as for real estate the picture appears to be quite different. The US index actually has one of the highest ES if measured with the Cornish fisher expansion. However, the discrepancy between the empirical and the parametric measure speaks for high measurement errors. This is further highlighted by the fact that the US index for real estate has the lowest VaR compared to the other country real estate indices (see Appendix H). Additionally, for some countries the ES measures for real estate are lower or quite similar compared to the local equity index. Therefore, it is ambiguous if an extension of a mixed-asset portfolio by international real estate would be beneficial in terms of tail risk.

In conclusion, it is questionable if global real estate can act as a risk-adjusted return enhancer. There are some indications that adding real estate country indices to a portfolio might help to decrease the tail risk of the overall mixed-asset portfolio. This effect would be strengthened if high global diversification benefits were to be found in the next subchapter. However, the evidence is rather weak and the risk of real estate is on average higher compared to the other asset classes. Nevertheless, less risk-averse investors were clearly able to generate higher absolute returns by allocating capital towards the Asia-Pacific real estate markets.

5.2.2 Diversification Benefits

Comparing Table 16 with Table 5 it looks like diversification benefits for a global broad index investing strategy are rather poor, as constant correlations are significantly higher compared to the national correlations. In particular, the correlation with the global bond index is around 0.4 higher.

Table 16:
Global – Constant Correlations between broad Indices

	GPR Gen. Qu.	**MSCI ACWI**	**Barclays Global Agg.**	**S&P Commodities**
GPR Gen. Qu.	1.00	0.80	0.42	0.22
MSCI ACWI	0.80	1.00	0.31	0.25
Barclays Global Agg.	0.42	0.31	1.00	0.18
S&P Commodities	0.22	0.25	0.18	1.00

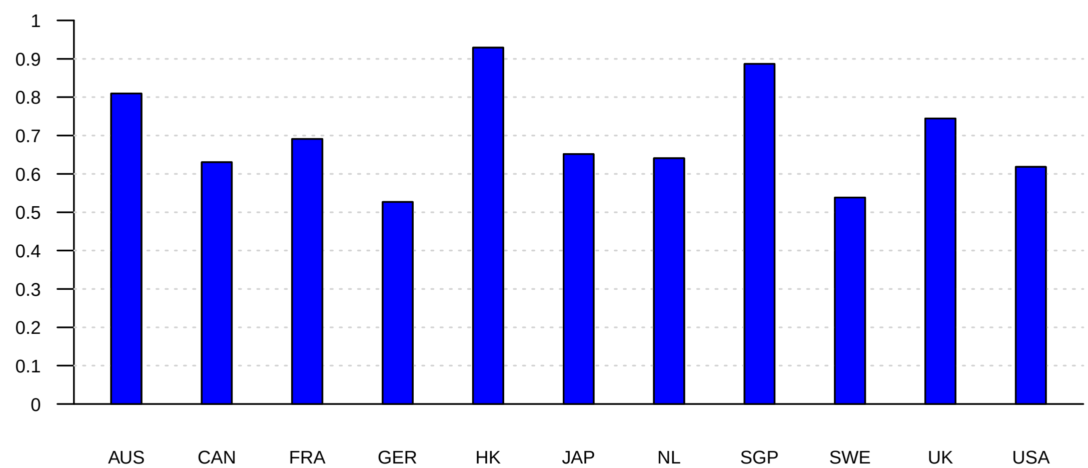
Notes: Monthly returns from the period 1990:1 to 2014:12 are used for the calculations.

Like for the national analysis, the correlation coefficients calculated by different methods are plotted against time in Fig. 22. The development of the coefficients follows patterns, which are closely related to the national development in Fig. 10. The financial crisis drove up all correlations implying diminishing diversification benefits. Nevertheless, like for the national analysis it is interesting to see that the correlation between real estate and equity decreased by 0.15 since the crisis and the DCC estimator also crossed the constant correlation from above at the beginning of 2013. In contrast, the desirable development from a diversification perspective cannot be found for the correlation with bonds. From a low of -0.03 since the crisis, all estimators steadily increased to a new maximum of over 0.6.

If investors do not rely on broad global index investing and the weights of the index, they can further diversify on a country level. This requires much more information in form of correlation coefficients. Table 17 shows only one third of the total correlation matrix neglecting correlations without real estate.[43] In the displayed quadrant a total of six negative coefficients in the bond quadrant can be identified. Furthermore, over 75 percent of the 341 correlations are below 0.5 indicating great international diversification possibilities with international real estate. Furthermore, it does not seem like the international real estate markets are as integrated as the international stock markets. In the pure real estate quadrant 76.4 percent of the coefficients are below 0.5 compared to only 25.5 percent of the coefficients in the equity quadrant and 48.9 percent of the coefficients in the bond quadrant.

However, several studies argue that local stock markets and the local real estate markets are cointegrated. Fig. 21 plots the country pairwise correlations between the national equity index and the respective local real estate index. Australia, Hong Kong and Singapore in particular reveal high coefficients over 0.8. This is not surprising as the stock markets of these three countries have a high real estate exposure (see Table 1). Nevertheless, the rest of the correlations are rather low with seven countries out of eleven having correlations below 0.7.

Fig. 21:
Global – Correlations between Real Estate and Equity within Countries

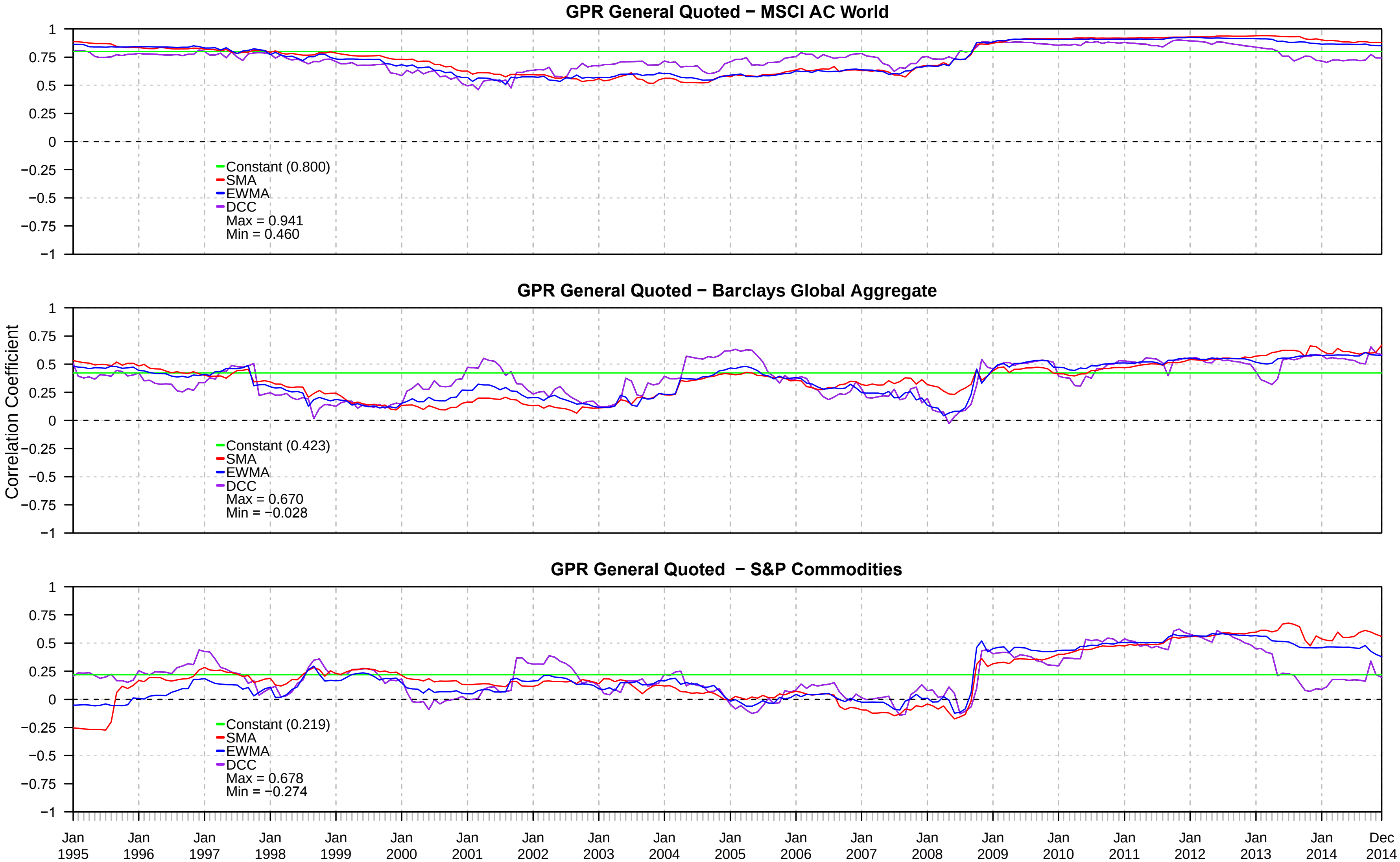


Notes: The graph depicts the local correlations between the MSCI World subindices and the GPR General Quoted subindices. The coefficients are based on monthly return data from 1985:1 to 2014:12.

The analysis suggests that investors should not rely on the market weights of the broad indices and therefore should not follow an investment strategy that solely replicates the global indices. Investors should rather try to mix a global portfolio based on broad country indices to be able to grasp additional and strong identified diversification benefits.

[43] The whole correlation matrix can be accessed via the offered website.

Fig. 22:
Global – Correlations with Real Estate over Time



Notes: The graph plots the correlation coefficients between real estate and the three asset classes. The calculations are based on monthly returns from 1990:1 to 2014:12. For the SMA a window of 15 years is used. The decay factor for the EWMA was set to 0.97 following the suggestion of RiskMetrics.

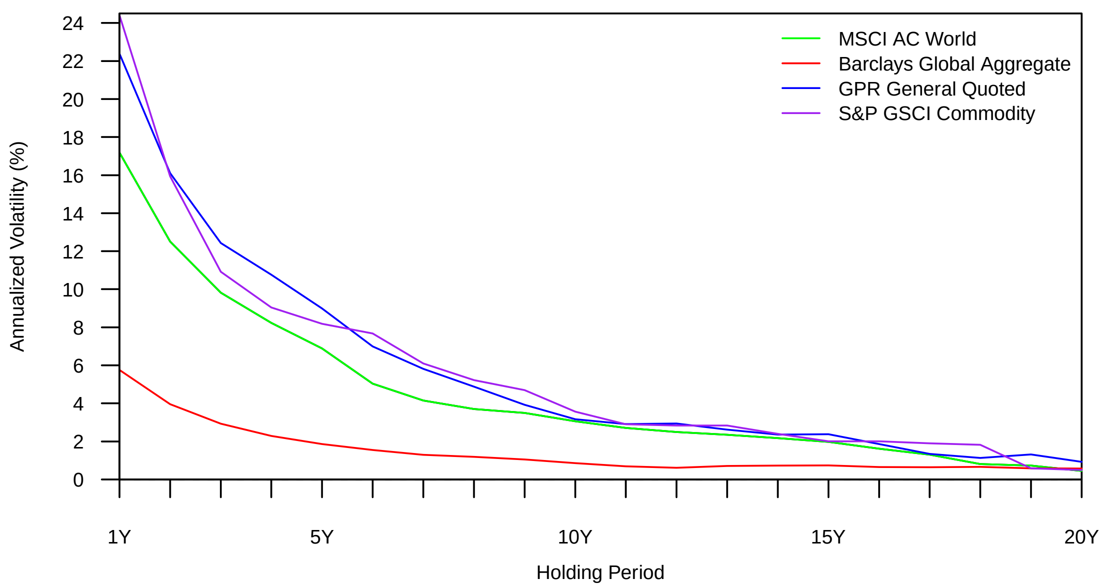
Table 17:
Global – Correlations with the GPR General Quoted Country Indices

		GPR General Quoted Indices										
		Australia	Canada	France	Germany	Hong	Japan	Netherlands	Singapore	Sweden	UK	USA
GPR General Quoted Indices	Australia	1.00	0.48	0.48	0.42	0.44	0.29	0.53	0.41	0.39	0.53	0.52
	Canada	0.48	1.00	0.41	0.33	0.35	0.32	0.44	0.35	0.36	0.42	0.50
	France	0.48	0.41	1.00	0.62	0.29	0.32	0.82	0.31	0.44	0.58	0.50
	Germany	0.42	0.33	0.62	1.00	0.26	0.24	0.64	0.26	0.33	0.48	0.45
	Hong Kong	0.44	0.35	0.29	0.26	1.00	0.16	0.31	0.67	0.23	0.33	0.33
	Japan	0.29	0.32	0.32	0.24	0.16	1.00	0.30	0.22	0.28	0.30	0.25
	Netherlands	0.53	0.44	0.82	0.64	0.31	0.30	1.00	0.35	0.46	0.58	0.52
	Singapore	0.41	0.35	0.31	0.26	0.67	0.22	0.35	1.00	0.22	0.37	0.42
	Sweden	0.39	0.36	0.44	0.33	0.23	0.28	0.46	0.22	1.00	0.39	0.38
	UK	0.53	0.42	0.58	0.48	0.33	0.30	0.58	0.37	0.39	1.00	0.55
	USA	0.52	0.50	0.50	0.45	0.33	0.25	0.52	0.42	0.38	0.55	1.00
MSCI Indices	Australia	0.81	0.50	0.44	0.42	0.51	0.28	0.49	0.51	0.40	0.51	0.47
	Canada	0.57	0.63	0.42	0.35	0.55	0.25	0.45	0.55	0.35	0.43	0.52
	France	0.49	0.43	0.69	0.50	0.40	0.30	0.61	0.40	0.35	0.49	0.47
	Germany	0.45	0.40	0.60	0.53	0.40	0.20	0.58	0.39	0.33	0.48	0.45
	Hong Kong	0.50	0.39	0.32	0.28	0.93	0.13	0.33	0.67	0.25	0.38	0.39
	Japan	0.35	0.36	0.31	0.23	0.28	0.65	0.30	0.34	0.33	0.34	0.35
	Netherlands	0.54	0.49	0.62	0.55	0.47	0.29	0.64	0.48	0.39	0.57	0.51
	Singapore	0.50	0.40	0.35	0.30	0.70	0.20	0.39	0.89	0.27	0.42	0.46
	Sweden	0.50	0.42	0.48	0.41	0.43	0.27	0.53	0.46	0.54	0.47	0.44
	UK	0.57	0.49	0.56	0.46	0.49	0.32	0.55	0.50	0.34	0.74	0.50
	USA	0.51	0.46	0.38	0.34	0.48	0.24	0.42	0.56	0.34	0.46	0.62
Citigroup Indices	Australia	0.73	0.37	0.43	0.35	0.34	0.23	0.44	0.32	0.33	0.40	0.33
	Canada	0.42	0.52	0.39	0.30	0.29	0.24	0.42	0.27	0.34	0.31	0.36
	France	0.26	0.15	0.63	0.44	0.11	0.22	0.57	0.10	0.18	0.35	0.17
	Germany	0.24	0.14	0.61	0.44	0.08	0.20	0.55	0.07	0.17	0.32	0.12
	Japan	0.03	0.01	0.25	0.14	0.06	0.34	0.18	0.07	0.06	0.15	-0.02
	Netherlands	0.26	0.14	0.61	0.45	0.10	0.20	0.57	0.08	0.18	0.33	0.14
	Singapore	0.51	0.36	0.51	0.42	0.39	0.32	0.54	0.45	0.47	0.42	0.39
	Sweden	0.48	0.31	0.55	0.41	0.25	0.26	0.53	0.27	0.32	0.39	0.30
	UK	0.26	0.21	0.39	0.30	0.09	0.29	0.36	0.04	0.12	0.49	0.11
	USA	-0.03	0.04	0.10	0.01	-0.08	0.10	0.03	-0.06	-0.02	0.00	0.05

Notes: The table presents the real estate quadrant of the correlation matrix of all considered country indices. The coefficients were calculated with monthly returns from 1985:1 to 2014:12.

In the following, the time diversification characteristics of real estate within a broad global index investment strategy are analyzed. All four indices inhibit decreasing risk with increasing holding periods (see Fig. 23). However, the global real estate index does not reflect superior characteristics and is clearly inferior to the global equity index for short and medium holding periods.

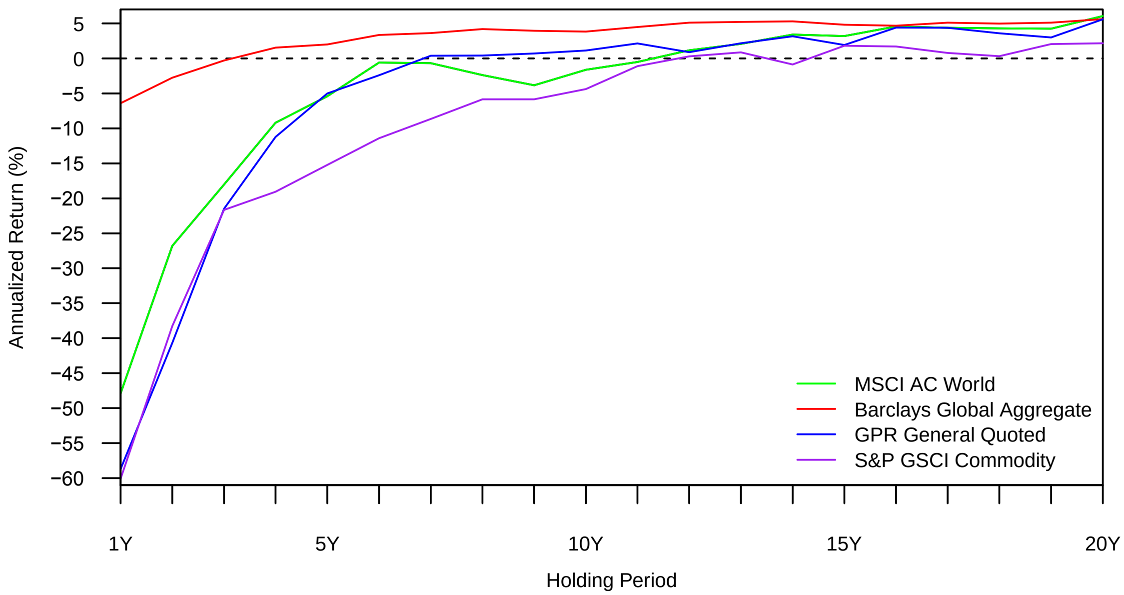
Fig. 23:
Global – Term Structure of Risk



Notes: The volatility calculations are based on monthly returns from 1990:1-2014:12. Furthermore, all potential starting points for the respective holding period are considered. Hence, the longer the holding period gets, the more difficult is a reliable calculation due to the limited observations.

In contrast Fig. 24 depicts that a real estate investment in terms of annualized returns was able to guarantee a slightly higher return for holding periods from seven up to eleven years compared to equity. However, an investment cannot beat the superior bond investment in terms of guaranteed returns over the analyzed sample period.

Fig. 24:
Global – Minimum annualized Returns depending on Holding Periods



Notes: The figure shows the minimum annualized return depending on the holding period over the period from 1990:1 to 2014:12. Furthermore, all starting points for the holding periods within the sample period are considered.

The tests for mean reversion in form of tests for stationarity neglect time diversification possibilities. For all indices the hypothesis of a unit root cannot be rejected as shown by the results in Table 18.[44]

Table 18:
Global – ADF and PP Tests

	MSCI ACWI	Barclays Global Aggregate	GPR Gen Q	S&P Commodities
Teststatistic	-0.12	-0.24	0.14	-2.28
Crit(1%)	-3.44	-3.44	-3.44	-3.44
Crit(5%)	-2.87	-2.87	-2.87	-2.87
Crit(10%)	-2.57	-2.57	-2.57	-2.57
PP Test (P-Value)	0.41	0.41	0.53	0.36

Notes: Scaled level data of the respective indices is used for the tests. For the ADF tests the AIC selection criteria is used to determine the number of lags. Furthermore, a drift component is added. Tests with an additional trend component have also been conducted but do not lead to different outcomes.

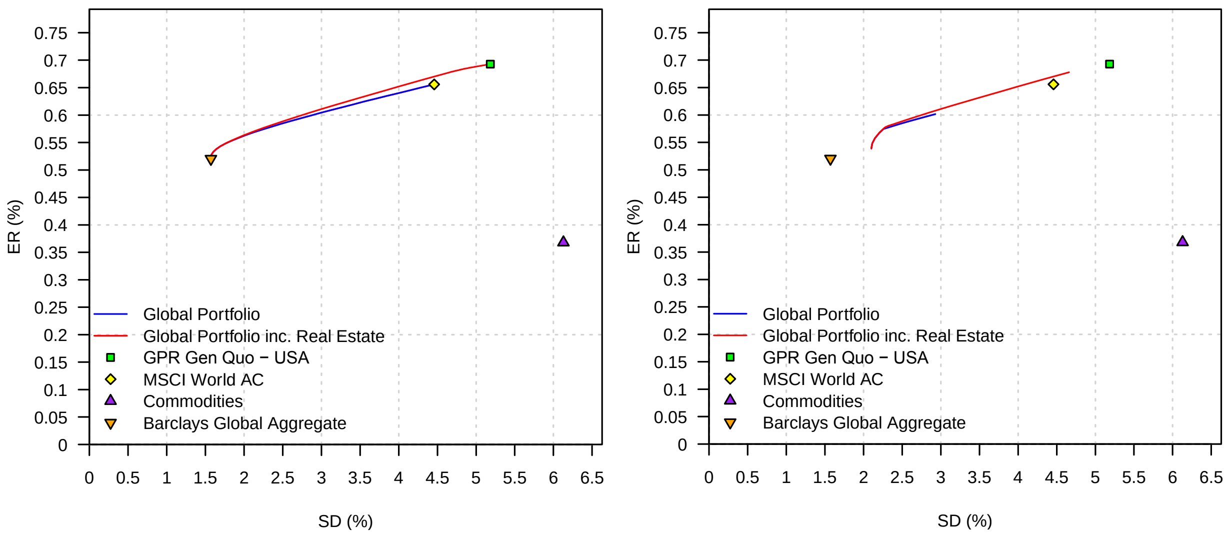
Neither the plotting exercises nor the tests for stationarity seem to ask for high allocations to the real estate index for time diversification purposes. However, low constant correlations (see Appendix F) and possible benefits in terms of quicker positive annualized returns of real estate compared to equity for mid-term investors (7-15 years) indicate minor advantages of an explicit real estate investment.

[44] Also for none of the GPR country indices indications for stationarity are found.

5.2.3 Portfolio Optimization

First, a mean-variance investor, who relies on a passive global index investing strategy, is assumed. The investor only allocates his money between the four broad global indices and does not consider country indices. Fig. 25 shows the frontiers from the mean-variance optimization. The right graph further considers a maximum allocation constraint of 60 percent per asset class. Comparing both graphs with the graphs from the national analysis (Fig. 12), confirms the results of the previous analyses. A global extension of a national portfolio by replicating global indices does not yield superior frontiers.

Fig. 25:
Global – Efficient Frontiers for broad Indices

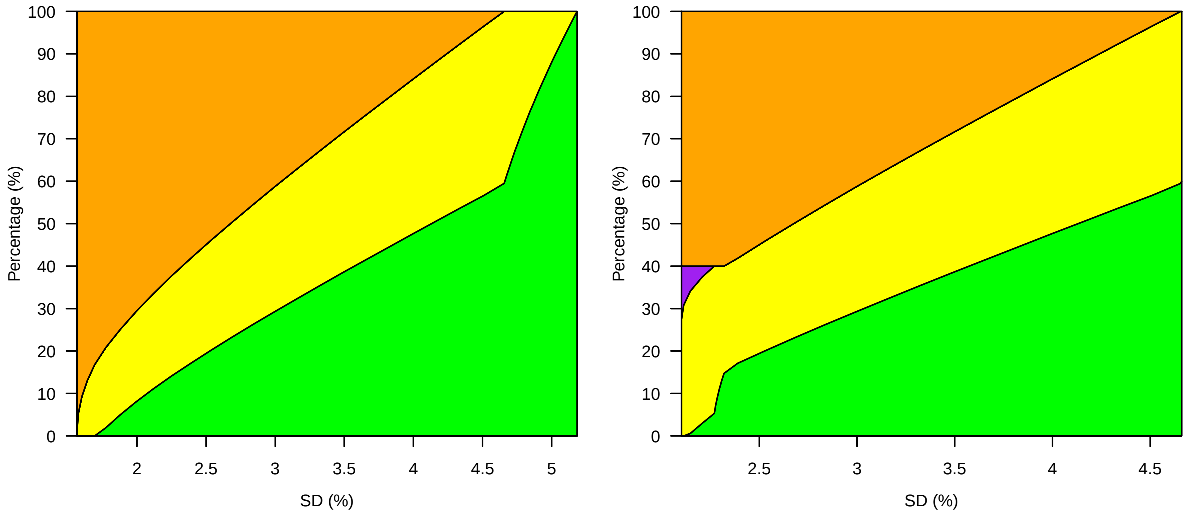


Notes: The figures depict the efficient frontiers for portfolios, which are exclusively invested in broad global indices. For all portfolios short selling is not allowed. For the right graph, the maximum allocation to one index is further limited to sixty percent. The optimization is based on monthly returns from 1990:1 to 2014:12.

Nonetheless, the weight allocation in lower risk areas is not negligible. At a SD of 2.5 percent, with the allocation constraint of 60 percent per asset, mean-variance optimization calls for almost 20 percent real estate (see Fig. 26). However, an inclusion of global real estate does not significantly improve the frontier in lower risk areas at first sight. This is substantiated by conducted spanning tests (not displayed). The frontiers further suggest that a global real estate investment may act as an absolute return enhancer allowing for higher expected returns like the previous observations suggested. This is also emphasized by the increasing weights along the frontier towards more risky portfolios (see Fig. 26).

However, as concluded in the previous analyses, an investor should rather try to optimize a portfolio based on country indices to generate an improvement from a pure national portfolio. Thus, the study pays more attention to the following analysis and bases further tests on portfolios, which consider country indices.

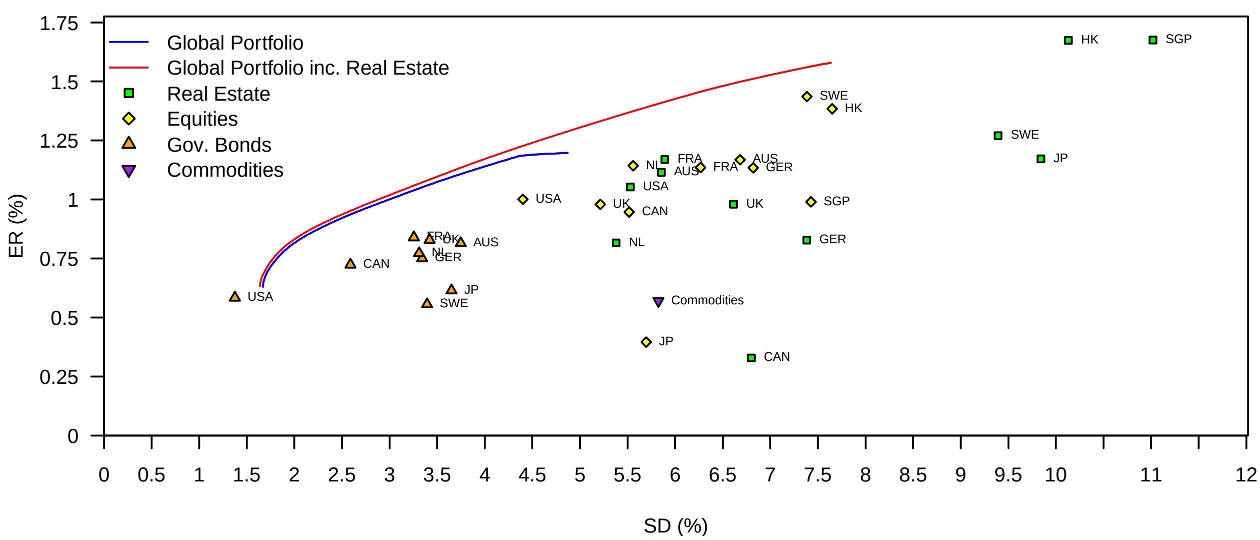
Fig. 26:
Global – Optimal Weights for broad Indices



Notes: The figures display the weight allocation corresponding to Fig. 25 dependent on SD.

All portfolios are constrained to a maximum allocation of 60 percent per asset class and country if possible. The constraints are implemented to avoid extreme weights and to vaguely approximate a more realistic strategic portfolio allocation. Further constraints within asset class portfolios are neglected in the following.

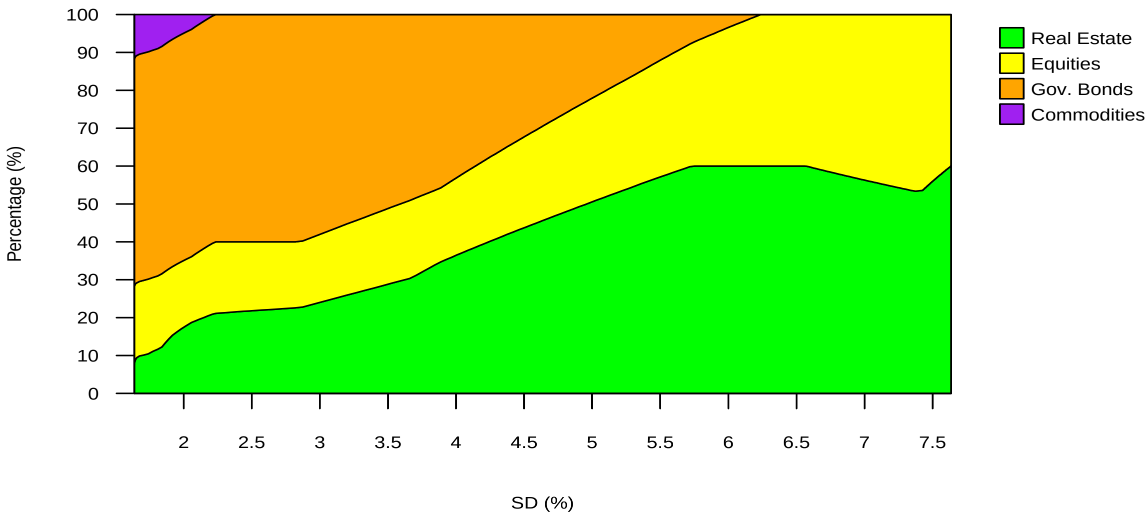
Fig. 27:
Global – Efficient Frontiers for Country Indices (A)



Notes: The graph plots the two efficient frontiers consisting of the displayed country indices and asset classes. For both frontiers several constraints are imposed. No short selling is allowed and the maximum allocation to one country and asset class is set to sixty percent. For the optimization monthly returns from 1985:1 to 2014:12 are used.

Fig. 27 shows the resulting frontiers for the international optimization based on country indices. The picture did not change much at first glance but this is only due to the scaling of the axis. It disguises an improvement along the whole frontier. The improvement is undermined by the fact that despite the orientation of the real estate indices towards the right upper corner, the minimum variance portfolio contains around seven percent real estate (see Fig. 28). This observation substantiates the correlation analysis of country indices, which indicated great diversification possibilities.

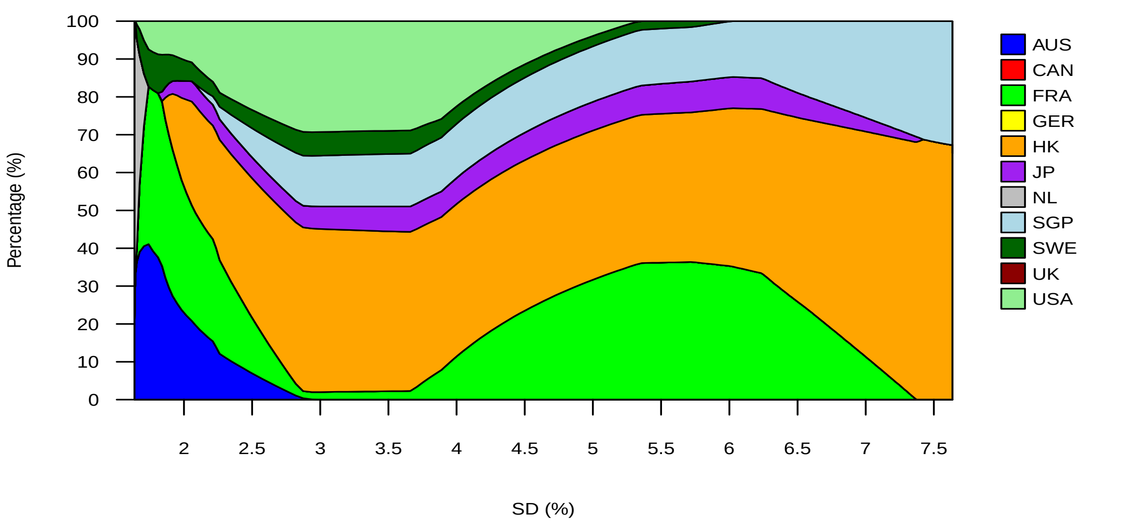
Fig. 28:
Global – Optimal Asset Class Weights for Country Indices



Notes: The graph shows the weights per asset class for all portfolios along the efficient frontier (corresponding to Fig. 27). The total weight per asset class equals the sum of all individual country weights of the respective asset class.

Fig. 29 gives a further impression of the country allocation of the isolated real estate portfolio, which is truly internationally diversified. The weighting towards local real estate is rather low ranging from zero up to a maximum of 30 percent.

Fig. 29:
Global – Optimal Country Weights of the Real Estate Portfolio



Notes: The graph shows the breakdown of the real estate portfolios within the mixed-asset portfolios corresponding to Fig. 27. The country weights are calculated by dividing the weight within the whole portfolio by the sum of all real estate investment weights.

Apparently, the higher the risk, the more weight is allocated to the highly volatile markets of Hong Kong and Singapore. This is not surprising and was indicated by the return enhancement analysis above.[45]

However, the results of the respective spanning tests in Table 19 challenge previous observations and curb the optimism. At all risk levels the intercepts and coefficients are not statistically significant. Even the higher risk portfolios are not statistically different. Thus, the extended portfolios do not seem to be able to yield a higher return than the benchmark portfolio at the same level of risk.

Table 19:
Global – Spanning Tests

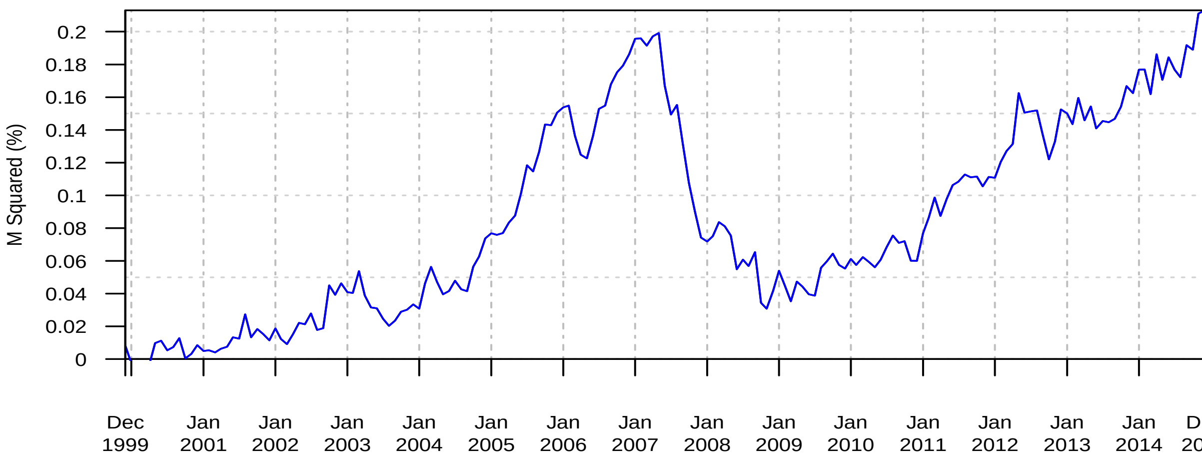
Portfolio	**Intercept**			**Benchmark Portfolio Coefficient**		
	Value	t-stat	Prob.	Value	t-stat	Prob.
MVP	0.000	0.878	0.381	0.983	-0.813	0.417
SD2	0.000	1.216	0.225	0.976	-1.002	0.317
SD2.5	0.000	0.818	0.414	0.985	-0.663	0.508
SD3	0.000	0.920	0.358	0.979	-0.944	0.346
SD3.5	0.001	0.909	0.364	0.971	-1.211	0.227
SD4	0.001	0.883	0.378	0.964	-1.207	0.228

Notes: The table presents spanning tests for optimized portfolios with different target SDs (corresponding to Fig. 27). The Portfolios are constrained to 60 percent per asset class and country.

As for the national analysis, M Squared of the theoretical portfolios with a target SD of two percent (Fig. 30) is plotted to evaluate potential time variation in benefits. M Squared for the global portfolios based on country indices evolved similiarly to the M Squared for national portfolios. The measure increased up to the beginning of the financial crisis and plummeted shortly afterwards. Even though it seems like adding real estate becomes more beneficial again, the upwards trend, beginning from the mid of 2009, is much more mildly compared to the national analysis. Nevertheless, a new height at the end of 2014 is reached, which is around ten basis points lower in contrast to the national analysis.

[45] The equity portfolio is very concentrated towards Swedish equity and might call for further constraints. However, a high concentration towards Swedish equities does not bias the real estate exposure through an explicit investment, as the MSCI Sweden index does not contain any real estate companies.

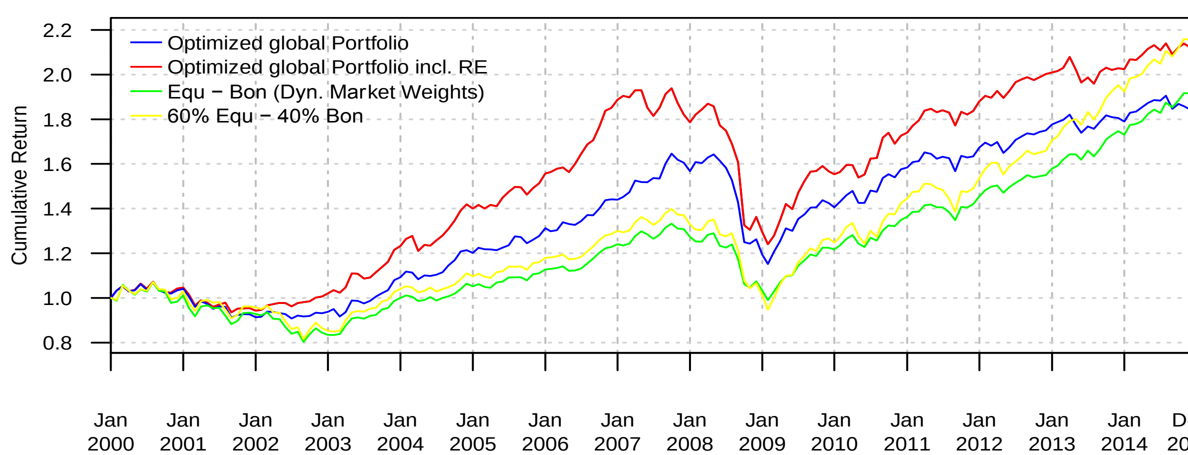
Fig. 30:
Global – Time-varying M Squared (A)



Notes: The figure shows M Squared comparing optimized global portfolios with and without a real estate investment. A portfolio manager with a target SD of two percent and the access to the individual country indices is assumed. The optimization process is conducted over a rolling 15 years window and is constrained to 60 percent per asset class. Moreover, short sales are not allowed and monthly returns from 1985:1 to 2014:12 are used.

Furthermore, the out-of-sample tests show that an investor, with a target SD of two percent, is able to generate an approximately 10 basis points higher average monthly return. Despite a similar increase of the SD of around 30 basis points, the return-risk ratio increases slightly. After equalizing the risk of the two portfolios an M Squared of only four basis points per month can be observed. The additional four percent higher ES of the portfolio with real estate challenges an extension. However, Fig. 31 depicts clear performance differences for different time frames.

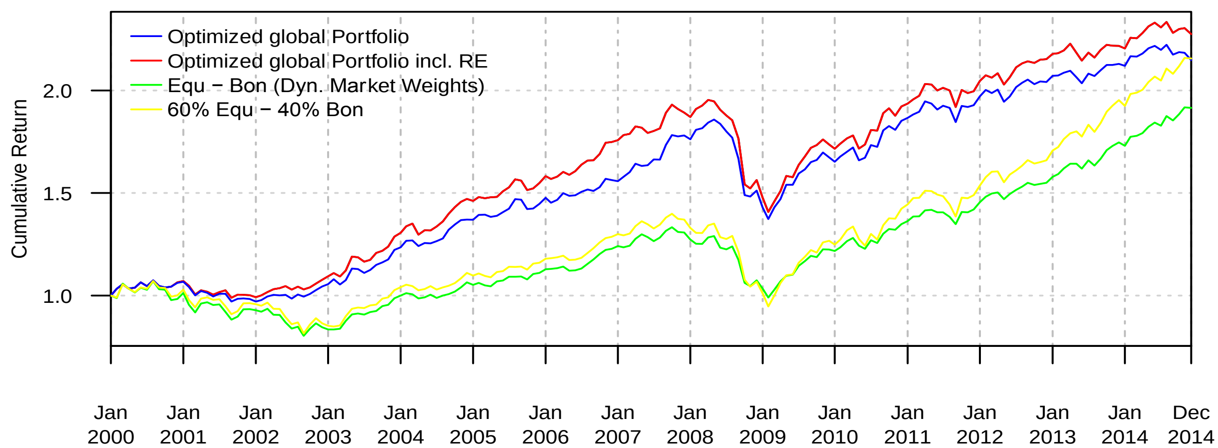
Fig. 31:
Global – Cumulative Returns of Optimized Portfolios (Fixed Window)



Notes: The red portfolio is rebalanced every month based on mean-variance optimization subject to a target SD of two percent. The portfolio represented by the blue line follows the same rebalancing strategy but has no access to international real estate. Short sales are not allowed, and the allocation to one asset class is limited to 60 percent. The optimization process is conducted over a rolling window of 15 years of monthly returns. The sample spans the period from 1985:1 to 2014:12

The out-of-sample tests have also been conducted in a different form. Instead of using a fixed rolling window, an expanding window is used. The portfolio manager would in general benefit from such an approach, as can be seen in Fig. 32. However, the performance difference between the portfolio with and without international real estate is even smaller with an M Squared of around one basis point. Hence, an extension is not economically beneficial (see Appendix J for detailed results).

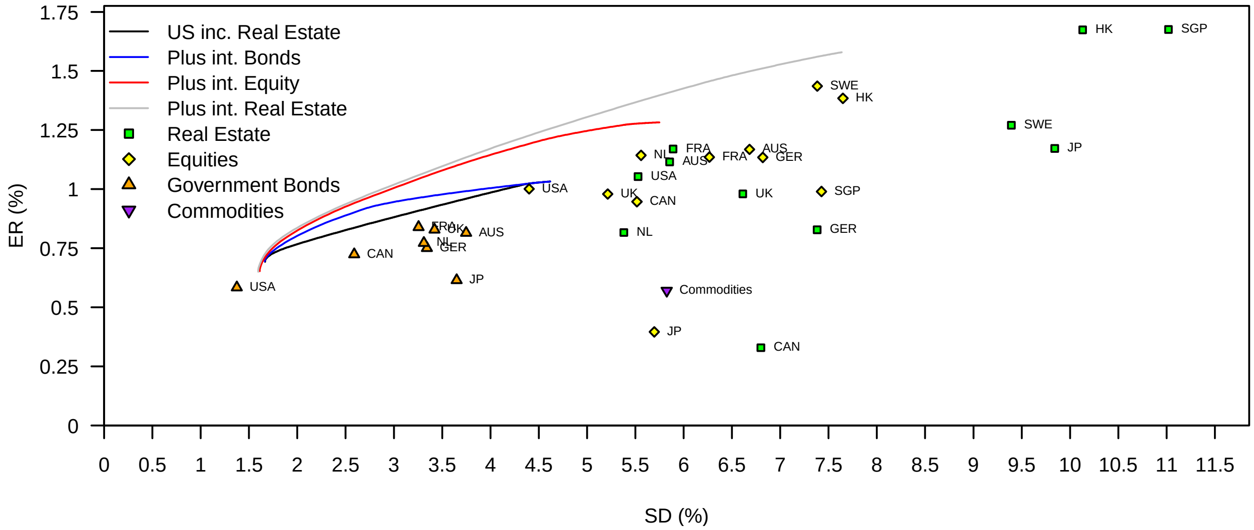
Fig. 32:
Global – Cumulative Returns of Optimized Portfolios (Expanding Window)



Notes: The red portfolio is rebalanced every month based on mean-variance optimization subject to a target SD of two percent. The portfolio represented by the blue line follows the same rebalancing strategy but has no access to international real estate. Short sales are not allowed and the allocation to one asset class is limited to 60 percent. An initial window of 15 years of monthly returns is used (1985:1-2014:12).

Finally, the study follows the methodology of Kroencke and Schindler (2012) in order to further test if a global extension of a national real estate portfolio within an internationally diversified portfolio should be considered. Fig. 33 gives a first impression of the static frontiers showing minor benefits in higher risk areas from extending the real estate portfolio to a global level. Spanning tests for different risk levels of the portfolios without (red Frontier) and with a global investment in real estate (grey Frontier) dispute a difference of the frontiers for portfolios with SDs up to 4.5 percent.

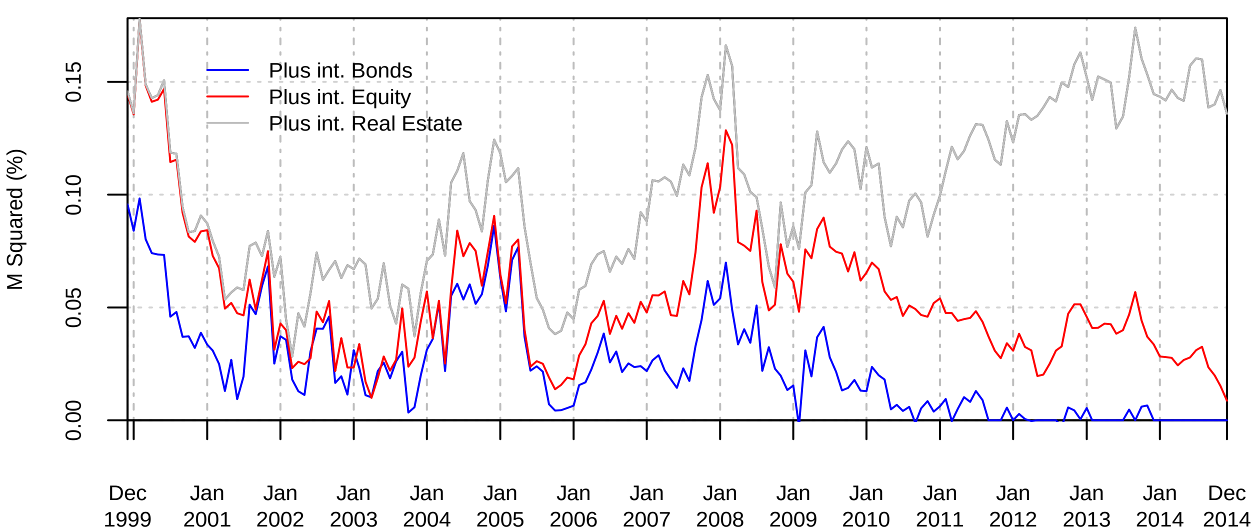
Fig. 33:
Global – Efficient Frontiers for Country Indices (B)



Notes: The graph shows four different efficient frontiers. All frontiers are based on the black frontier but contain additional international investments. The allocation to one asset class is restricted to sixty percent. Furthermore, short sales are prohibited. For the optimization process monthly returns from 1985:1 to 2014:12 are used.

Fig. 34 confirms this observation for the analysis across time. Even though the gap starts to widen since 2009, M Squared counts for only around 10 basis points per month at the end of 2014. In combination with the higher tail risk of most real estate investments the improvement is small.

Fig. 34:
Global – Time-varying M Squared (B)



Notes: The graph shows the performance difference of dynamic mean-variance optimized portfolios subject to a target SD of two percent. Short sales are not allowed and the allocation to one asset class is limited to 60 percent. The optimization process is based on a rolling window of 15 years. Monthly returns from 1985:1 to 2014:12 are used.

Comparing the out-of-sample exercises yields interesting results. Adding cross-border real estate investments to a global portfolio with national real estate does not significantly increase the ES and the SD but increases the expected return at the same time. This leads to an average M Squared of 7 basis points (fixed window) and 1.5 basis points (expanding window). Intriguingly, this is slightly higher than the M Squared from switching from a portfolio completely without real estate to a portfolio with global real estate (fixed 4 basis points and expanding 1 basis point). This suggests high and unpredictable risk of a real estate investment in the US. When targeting different SDs, similar results to the national out-of-sample exercises are obtained. However, the highest mean return can be reached, when targeting a SD of two percent. In general, the results of the out-of-sample tests do not advocate an extension of a global portfolio (see Appendix J for details).[46]

In total, evidence for benefits of investing in global real estate are mixed and no clear recommendation for adding global real estate to a global mixed-asset portfolio can be given. On the one hand, the promising diversification benefits are substantiated by the conducted analyses across time and clear allocations to real estate in low risk portfolios. On the other hand, the spanning and out-of-sample tests challenge an extension.

[46] The corresponding weights for the time-varying analysis can be found in Appendix K.

5.3 Discussion

This section gives a brief overview of the comprehensive empirical results of chapter five and positions them within the outcomes of previous research. The national and global outcomes for each of the three potential benefits are summarized and complemented by the portfolio optimization exercises. Finally, a consolidating recommendation for investors and regulators is given.

Return enhancement

On the one hand, the empirical tests suggest that an inclusion of real estate in a portfolio in terms of risk-adjusted returns is questionable due to its high volatility and downside risk. On the other hand, an explicit allocation might help less risk-averse investors to enhance their total return. These outcomes are in line with most of the discussed studies (Idzorek *et al.*, 2006; D. Brounen & Eichholtz, 2003; Hoevenaars *et al.*, 2008; Ling & Naranjo, 2002).[47] In particular, real estate indices in the Asia-Pacific region displayed high mean returns and should help to enhance the total return of a portfolio. In contrast, a pure national real estate investor has only the possibility to benefit from a slightly higher mean return of real estate. Intriguingly, the results in form of tail risk measures point out that there might be the possibility for investors to decrease the downside risk of their portfolio through an allocation to different real estate markets. However, the possibility has to be seen with caution as the out-sample tests highlight the difficulty to predict extreme negative returns and differences in tail risk measures indicate biased results.

Risk Diversification

The analysis revealed promising results with low constant and also time-varying correlation coefficients between national indices and also country indices. Investors can benefit from getting exposure to unique risk factors of real estate. This is highlighted by the global mean-variance optimization exercise with country indices. Despite the high volatility of real estate it already calls for allocations within low risk portfolios. This observation is undermined by the identified global diversification benefits within a real estate portfolio. Real estate markets do not appear to be as cointegrated as the equity markets. This substantiates the observations of Eichholtz *et al.* (1998), Ciochetti *et al.* (2015), and Pavlov *et al.* (2015). However, similar to the study of Lu *et al.* (2013), the results point out decreasing diversification benefits in form of higher correlations on a national and on a global level during times of high volatility. Time-varying correlations skyrocketed during the financial crisis and the time varying M Squared measures plummeted accordingly. Differences between equity returns and real estate returns seemed to vanish distorting the intended risk diversification and highlighting potential tail dependencies. Equally to several studies before it becomes clear that investors have to pay close attention to trends in diversification benefits (see Kroencke & Schindler, 2012; Chong *et al.*, 2009; Case *et al.*, 2012; Lizieri, 2013; Liow *et al.*, 2015). In particular, investors have to pay close attention to the recent

[47] Intriguingly, Hudson-Wilson *et al.* (2005) find the opposite for their mingled real estate index. This should be due to the high percentage of private debt and CMBS and the appraisal based private equity real estate index.

decrease of some correlation indicators, which might indicate strengthening diversification possibilities of a real estate investment.

Another discussed dimension of diversification is time diversification for investors with longer holding periods. As returns do not seem to be independent over time, the topic should play an important role and should become more sizable with expanding time series data. On the one hand, no strong indications for mean reversion of the national and global indices in form of tests for stationarity can be identified. This is similar to the studies of Fugazza *et al.* (2009) and Stevenson (2002). On the other hand, the plotting exercises (especially for national investors) indicate that investors with mid-term horizons between five and fifteen years are able to take advantage of an explicit allocation to real estate. The decreasing volatility of annualized returns is below or at least similar to an equity investment within this range. Furthermore, real estate was able to generate a guaranteed positive annualized return quicker (7 years) compared to an equity (11 years) and commodity (12 years) investment. However, the results should be interpreted carefully due to the limited length of the sample. Moreover, real estate investors have to consider higher reinvestment risk due to the distribution requirements of REITs and therefore high interim cash flows. The results are mixed but highlight some new insights in form of potential advantages for real estate investors with longer investment horizons.

Inflation Hedging

Like in previous studies negative and insignificant coefficients for expected inflation and realized inflation are identified for the short-term analysis (e.g. Liu *et al.*, 1997). However, coefficients mainly turn positive after controlling for economical variables in the used regressions. This is in line with the proxy hypothesis (Fama & Schwert, 1977) and the study of Hoesli *et al.* (2008). Hudson-Wilson *et al.* (2005) would expect good short-term hedging capabilities for REITs if the market for listed real estate evolved independently from the stock market. Intriguingly, an investment seemed to be able to provide some hedging capabilities against expected inflation between 2000 and 2007, a time period with low correlations (see Fig. 10). Furthermore, any chance of hedging capabilities seemed to disappear with increasing correlations after the financial crisis, when correlations were at high levels. However, when analyzing the whole sample period the capabilities to protect against realized, expected or unexpected inflation in the short run are weak. The recent study of C. L. Lee and Lee (2014) also cannot find evidence for different European countries adding some cross country robustness to the results. Only an investment in the S&P GSCI commodities is a superior tool to hedge a portfolio against overall inflation risk in the short run. Even a bond investment displays only perverse hedging capabilities against unexpected inflation.

However, the picture changes when analyzing the long-term hedging capabilities. Similar to the studies of Hoesli *et al.* (2008), Maurer and Sebastian (2002), and C. L. Lee and Lee (2014), the results suggest hedging capabilities of listed real estate in the long run. The error correction model indicates a significant long-term equilibrium relationship between listed real estate and expected and realized inflation. Besides listed real estate, only bonds display an equilibrium relationship. Against expectations, a commodity investment cannot retain its great hedging capabilities in the long run. Thus, based on the empirical tests an investment in real estate is especially superior to a competing equity investment and may help investors to protect their portfolio better against inflation.

In conclusion, the results highlight that an investment in listed equity real estate entails several benefits but also has many shortcomings (for a summary table of pros and cons see Appendix L). Thus, it is difficult to give a clear answer to the question if a well-diversified investor should allocate funds to an explicit real estate investment. Nevertheless, the findings have several implications for investors. Three possible benefits have to be highlighted. First, investors can get an exposure to unique risk factors and therefore further diversify their portfolios. In particular, an international diversified real estate portfolio is promising. Second, venturesome investors may enhance their absolute return with an investment. Third, investors with longer holding periods are able to benefit in form of inflation hedging and time diversification. However, despite the mentioned advantages investors should not ignore the high tail risk and volatility of real estate. The possibility of high tail dependency with other asset classes should especially be kept in mind. Furthermore, investors should be warned by the spanning and out-of-sample tests, which mainly dispute potential benefits of adding a real estate investment (for summaries see Appendix L).

Regulators can see a positive development towards more transparent and liquid real estate markets, which might mitigate the possibility of future severe market crunches. However, such a development should not come with the cost of bigger deviations from the underlying real estate market. Therefore, they have to prevent a cointegration with the equity market. They have to observe and limit the possibility that firm characteristics highly influence the value of the companies. Executives should not be able to deviate from a prudent property investment strategy. Furthermore, a key element for regulators should remain the education of investors and investment consultants about the unique characteristics of real estate companies. Having this agenda in mind, listed equity real estate based on the empirical results should be further promoted as a unique asset class.

6 Concluding Remarks

This study aimed to answer the question if an index investor should include an explicit investment in listed equity real estate to a mixed-asset portfolio. Thus, the study gives a comprehensive overview about the return enhancement, diversification and inflation hedging capabilities of listed equity real estate compared to the other asset classes. Besides the individual analysis, combining portfolio optimization exercises are conducted. Furthermore, the study enables the reader to separate between a global and a purely national diversified US investor. Hence, the study connects and expands the ideas of previous research and thus contributes to a better understanding of listed equity real estate as an alternative asset classes.

The results of the empirical tests in chapter 5 are mixed, but several conclusions can be drawn. In general, the empirical tests indicate strong diversification benefits for the whole sample period and different sub periods. However, investors have to be aware of tail dependencies with other asset classes as indicated by the increasing correlations during the financial crisis in 2008. Furthermore, high volatility and high tail risk of real estate contradict strong return enhancement capabilities in form of risk-adjusted returns. Even high mean returns, which only help venturesome investors to boost their total return, cannot compensate the high risk. These critical characteristics of an investment are also highlighted by the optimization exercises. Spanning and out-of-sample tests are in most cases not able to identify statistical differences between portfolios with and without listed equity real estate. Nevertheless, the time-varying analysis emphasizes major differences in different time periods. Additionally, the clear allocations to real estate in low risk portfolios still indicate the diversification possibilities of real estate. For long-term investors the empirical results further point out inflation hedging capabilities, indications for time diversification can be identified as well. Overall, the empirical findings highlight the importance for investors to regularly reassess the benefits of listed equity real for their particular needs. Furthermore, the mixed results emphasize the importance of the big picture before drawing irrational conclusions based on only one of the three investigated questions.

Despite the wide range of the empirical tests and broad analysis, three important limitations of the conducted study have to be reflected. First, most of the methodologies do not consider different investor horizons. This is critical, as returns do not seem to be independent across time. Different horizons and lifecycle asset allocation strategies lead to deviating results. However, such tests are still very restricted by the limited length of the available time series data. Second, the portfolio optimization technique does not properly consider extreme value theory and the need of investors to further limit downside risk. Thus, frameworks, that better accommodate tail dependencies and tail risk, might lead to different allocations. Another problem of the optimization technique is the need for proper expectations of investors as input factors. In this context, the growing market for real estate specific derivatives might allow for testing out-of-sample investment strategies, which use implied volatilities to better reflect investors' expectations. Third, the broad indices do not provide details about different types of real estate. Therefore, applying the empirical analysis to subcategories of commercial real estate would lead to further implications for investors and regulators. In general, it is still difficult to draw robust conclusions based on the limited data availability. However, longer time series data and the promotion of real estate to a new headline sector of the GICS should encourage more research about the role of listed equity real estate in a mixed-asset portfolio.

Ultimately, the study can contribute to the literature by equipping investors and regulators with a comprehensive overview and new insights into the role of listed equity real estate as an alternative asset class. On the one hand, the study highlights several desirable characteristics of an investment like diversification, absolute return enhancement and long-term inflation hedging capabilities. On the other hand, investors should stay cautious, as an investment displays weak risk-adjusted return enhancing capabilities, high tail risk, and tail dependencies with other asset classes. Therefore, an answer to the outlined research question highly depends on the needs and the investment horizon of an investor. However, investors should pay close attention to the evolving sector and be aware of its unique characteristics as a distinct asset class.

Appendix

Appendix A – Data Sources

Table A:
Overview of the used Data

	Name	Asset Class	Currency	Data available from	Provider	Vendor
Countries	GPR Country Indices	Real Estate	Local/USD	12/31/1984	GPR	GPR
	EPRA/NAREIT	Real Estate	Local/USD	12/31/1990	FTSE	Datastream
	MSCI Country Indices	Equity	Local/USD	12/31/1969	MSCI	Datastream
	Citigroup WGBI Subindices	Government Bond	Local/USD	12/31/1984	Citigroup	Datastream
	US Aggregate Index	Bonds	USD	01/01/1976	Barclays	Datastream
Global	GPR General	Real Estate	USD	12/31/1983	GPR	GPR
	GPR General Quoted	Real Estate	USD	12/31/1983	GPR	GPR
	GPR 250	Real Estate	USD	12/31/1989	GPR	GPR
	GPR 250 Reits	Real Estate	USD	12/31/1989	GPR	GPR
	MSCI World	Equity	USD	12/31/1969	MSCI	Datastream
	MSCI All Country World	Equity	USD	12/31/1987	MSCI	Datastream
	Barclays Global Aggregate	Bonds	USD	01/31/1990	Barclays	Datastream
	S&P GSCI Commodity	Commodities	USD	12/31/1969	S&P	Datastream
Inflation Analysis	Real Gross Domestic Product-USA	-	USD	12/31/1946	US. Bureau of Economic Analysis	FRB of St. Louis
	Industrial Production Index-USA	-	USD	12/31/1918	Board of Governors of the FRS	FRB of St. Louis
	M2 Money Stock	-	USD	12/31/1958	Board of Governors of the FRS	FRB of St. Louis
	CPI for All Urban Consumers: All Items	-	USD	12/31/1912	US. BLS	FRB of St. Louis
	CPI for All Urban Consumers: Rent of primary Residence	-	USD	12/31/1939	US. BLS	FRB of St. Louis
	CPI for All Urban Consumers: Owners' equivalent rent	-	USD	12/31/1981	US. BLS	FRB of St. Louis
	3 Month Libor	Treasury Bill	USD	01/31/1971	US Federal Reserve Data Releases	Quandl.com
	3 Month Treasury Bill USA	Treasury Bill	USD	12/12/1953	US Federal Reserve Data Releases	Datastream
Others	Risk Loadings &	-	USD	01/07/1926	Kenneth R. French Website	WRDS
	1 Month Treasury Bill USA	Treasury Bill	USD	01/07/1926	Kenneth R. French Website	WRDS

Appendix B – Time-varying Weights of optimized Portfolios (National)

The graphs below depict the asset class weights based on the portfolio optimization exercises in chapter 5.1.3. The optimization process is subject to a two percent target SD. Moreover, short selling is forbidden and weights are constrained to 60 percent per asset class and country. The initial window is set to 15 years. Furthermore, Fig. A weights stem from a optimization over a rolling fixed window, whereas Fig. B weights come from an optimization procedure with an expanding window.

Fig. A:
National – Weights of optimized Portfolios (Fixed Window)

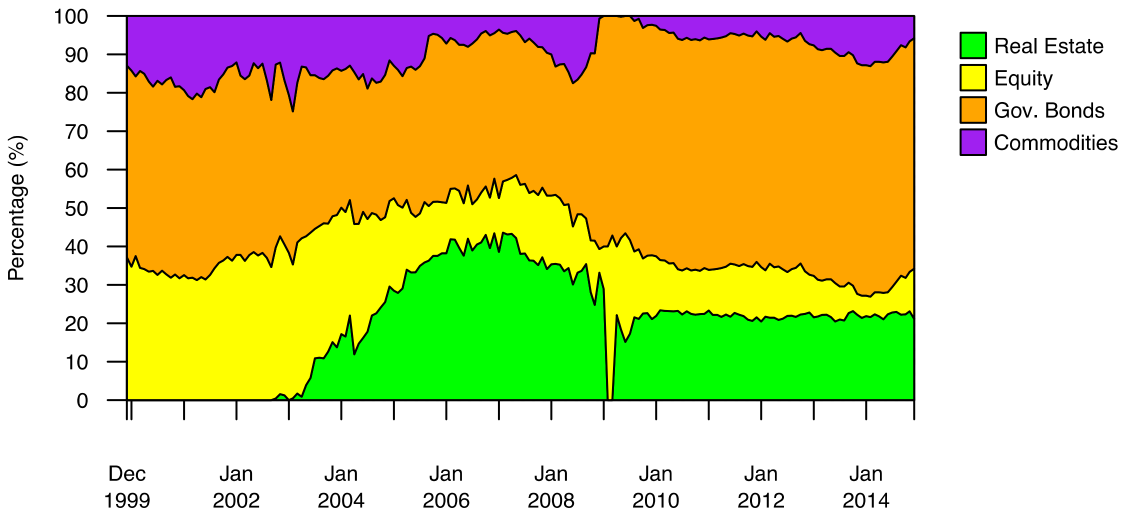
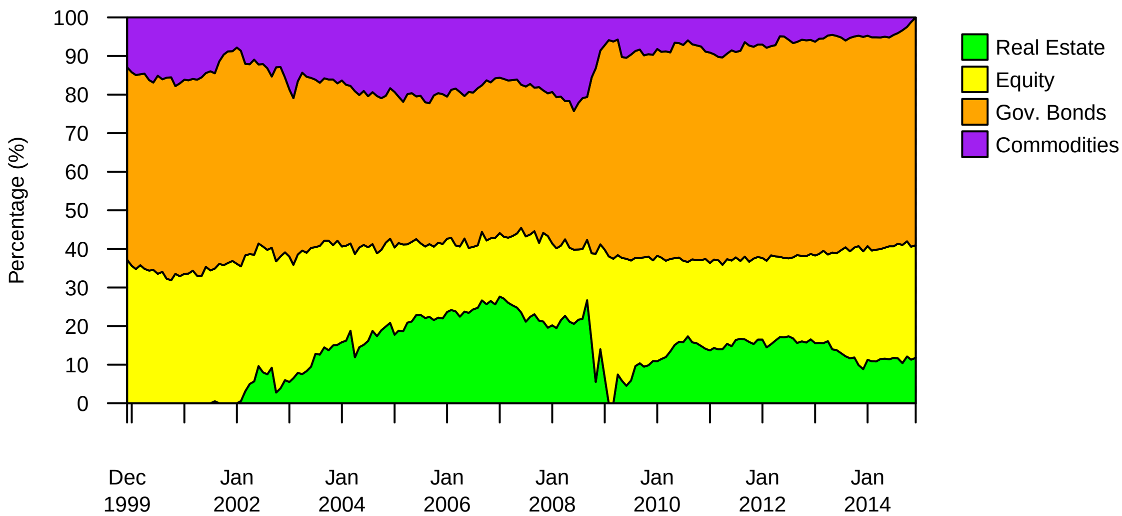


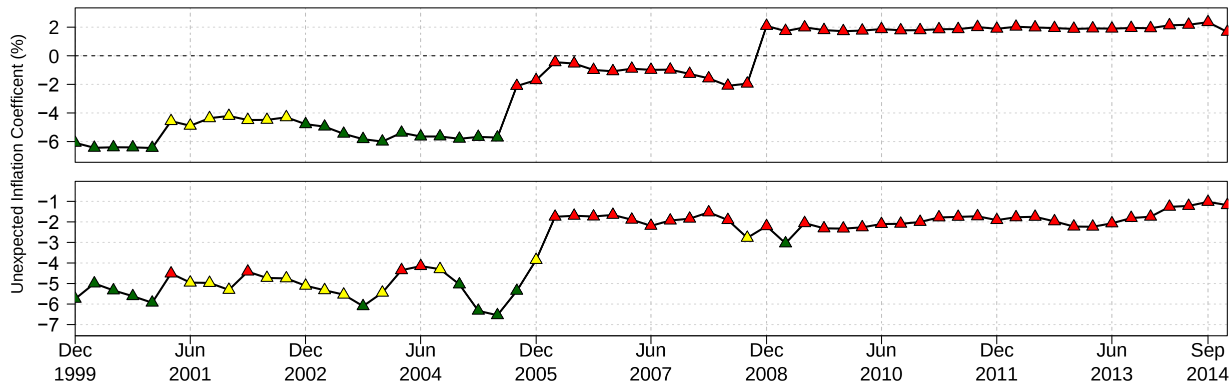
Fig. B:
National – Weights of optimized Portfolios (Expanding Window)



Appendix C – Rolling Inflation Hedging Regressions

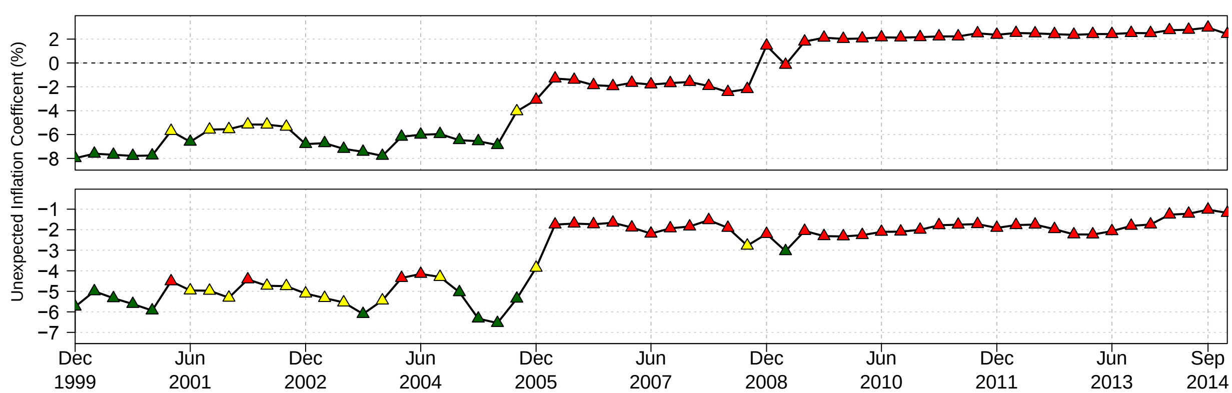
The tables below reveal further details of the inflation hedging capabilities over different time frames and should be taken as a supplement for the analysis in chapter 5.1.4. Fig A. and Fig B. depict the coefficients of the realized inflation and unexpected inflation based on rolling regressions over a fixed 15-year window. The coefficients evolve quite similar over time and are mostly negative. As unexpected inflation explains most of the fluctuation of the realized inflation, this observation is not surprising.

Fig. A:
Total Inflation Hedging – Rolling Regression



Notes: The graphs show the point estimates for the coefficients of the total realized inflation rate and the real estate return as the dependent variable. In contrast to the upper graph, the graph below shows the results for the coefficients after controlling for macroeconomic variables. Moreover, the regressions are run over a fixed window of 15 years and quarterly returns are used. Green stands for significant at a 5 percent level, yellow at a 10 percent level, and red above the 10 percent level.

Fig. B:
Unexpected Inflation Hedging – Rolling Regression



Notes: The graphs show the point estimates for the coefficients of the unexpected inflation rate and the real estate return as the dependent variable. The graph below shows the results for the coefficients after controlling for macroeconomic variables. The regressions are run over a window of 15 years and quarterly returns are used. Green stands for significant at a 5 percent level, yellow at a 10 percent level, and red above the 10 percent level.

Appendix D – ADF and PP Tests as Part of the Inflation Analysis

Preliminary to the inflation analysis in chapter 5.1.4, tests for the order of integration of the considered time series reveal that Libor and Treasury Bill Rate indices might not be I(1). Only the ADF test for the Libor series indicates that it is I(1) with a confidence level of 5 percent. This might lead to spurious regression coefficients. Intriguingly, if both variables are excluded, the coefficient for expected inflation in the real estate level regression will become insignificant. If only one of the variables is excluded the coefficient will stay significant. However, as indicated by Glascock *et al.* (2002) there should be a close link between interest rates and real estate returns, which speaks against an exclusion and for the used framework.

Table A:
National – Inflation – ADF and PP Tests

	I0		I1	
	ADF	**PP Test**	**ADF**	**PP Test**
Real Estate			***	***
Equity			***	***
Bonds	**		***	***
Commodities			***	***
MSCI World		*	***	***
Industrial Production USA			***	***
M2			***	***
Libor			**	
Treasury Bill 3M				
MSCI USA			***	***
Real GDP USA			***	***
Realized Inflation			*	**
Expected Inflation			*	***
Unexpected Inflation	***	***	***	***

Notes: The asterisks indicate if the null hypothesis of a unit root is rejected. The number of asterisks show the statistical significance at a confidence level of: * 10%, ** 5%, and *** 1%. The tests are conducted with quarterly returns over the whole sample period (1985:1-2014:12). For the ADF test a drift component is included.

Appendix E – ECM with different CPI Subindices

The table below presents the ECM results for two CPI subindices, which should be closely related to commercial rents. The intuitive relation is confirmed by the empirical results. They point out that a real estate investment has hedging abilities against price movements of the two indices in the long run but also in the short run. The coefficient of expected inflation is for all four regressions positive and at least weakly significant at a ten percent level. Hence, despite the low error correction term investors should benefit from good hedging capabilities against expected inflation. Nevertheless, hedging capabilities against unexpected inflation cannot be identified.

Table A:
Error Correction Model with different CPI subindices

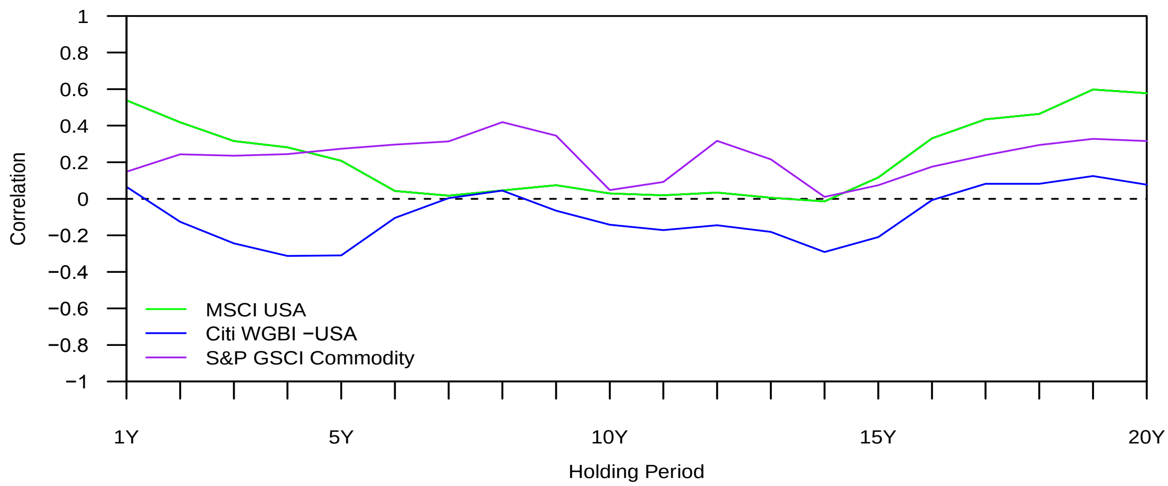
	Used Inflation Subindex					
	Owners' equivalent rent			Rent of primary residence		
Levels	Coeff.	Prob.		Coeff.	Prob.	
Constant	0.564	0.052	*	0.580	0.121	
Expected Inflation	8.231	0.044	**	9.487	0.099	*
Unexpected Inflation	7.788	0.118		8.798	0.170	
MSCI World	0.522	0.018	**	0.194	0.675	
MSCI USA	-0.114	0.657		0.102	0.728	
Industrial Production USA	-0.459	0.707		-0.351	0.797	
M2	-1.315	0.055	*	-2.961	0.075	*
Real GDP USA	3.650	0.013	**	5.027	0.000	***
Libor	-8.963	0.000	***	-3.076	0.259	
Treasury Bill 3 Month	-0.386	0.845		-3.404	0.346	
Adjusted R squared:	0.977			0.975		
First Differences	Coeff.	Prob.		Coeff.	Prob.	
Constant	0.014	0.356		0.006	0.730	
Expected Inflation	6.505	0.095	*	7.146	0.030	**
Unexpected Inflation	-0.836	0.671		1.241	0.799	
MSCI World	0.202	0.182		0.140	0.361	
MSCI USA	0.549	0.005	***	0.597	0.002	***
Industrial Production USA	-0.596	0.397		-0.263	0.683	
M2	-2.253	0.028	**	-2.717	0.017	**
Real GDP USA	1.565	0.234		2.204	0.097	*
Libor	-12.507	0.040	**	-9.551	0.081	*
Treasury Bill 3 Month	2.726	0.559		1.457	0.726	
ECM	-0.258	0.001	***	-0.244	0.002	***
Lag1	0.283	0.001	***	0.261	0.000	***
Lag2	0.024	0.693		0.002	0.968	
Adj. R squared	0.542					

Notes: The table presents regression results for the error correction model (equations 26 and 27). Quarterly logged and scaled data from 1985:1 to 2014:12 is used. The dependent variable is the GPR General Quoted subindex for the US.

Appendix F – Constant Correlations depending on Holding Periods

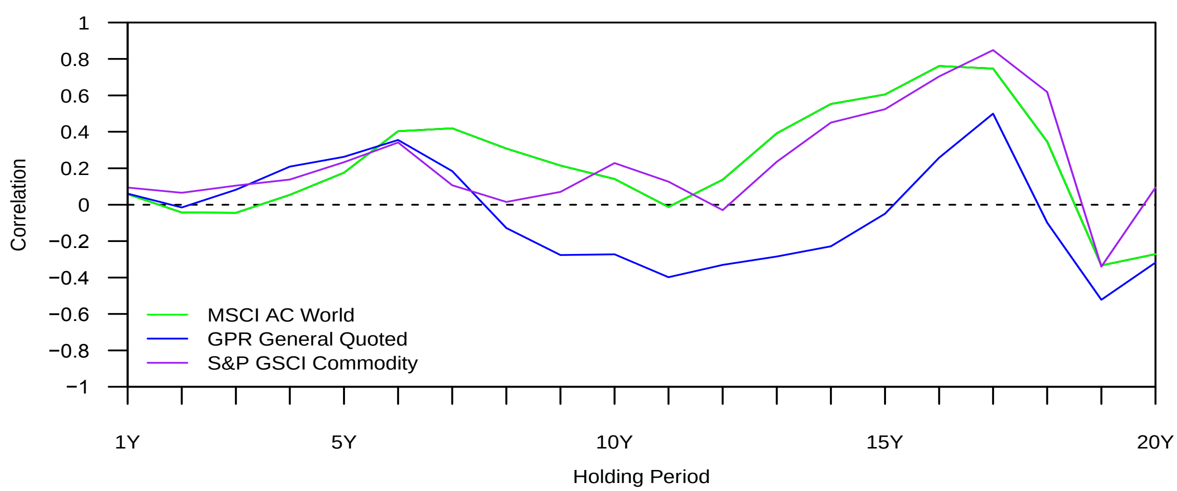
If returns are not idependently distributed accross time, correlation coefficients can differ accross holding periods. This would imply diversification benefits, which depend on investors investment horizons. The figures below point out that constant correlations indeed vary significantly with increasing holding periods. From an investors point of view, who is seeking great diversification benefits with an additional real estate investment, holding periods between ten and fifteen years seem to be most promising.

Fig. A:
National – Correlations between Indices



Notes: The graph shows constant correlations of the GPR Gen Q-USA with the other national indices, depending on the holding period of investors. Monthly return data from 1985:1 to 2014:12 is used.

Fig. B:
Global – Correlations between broad Indices

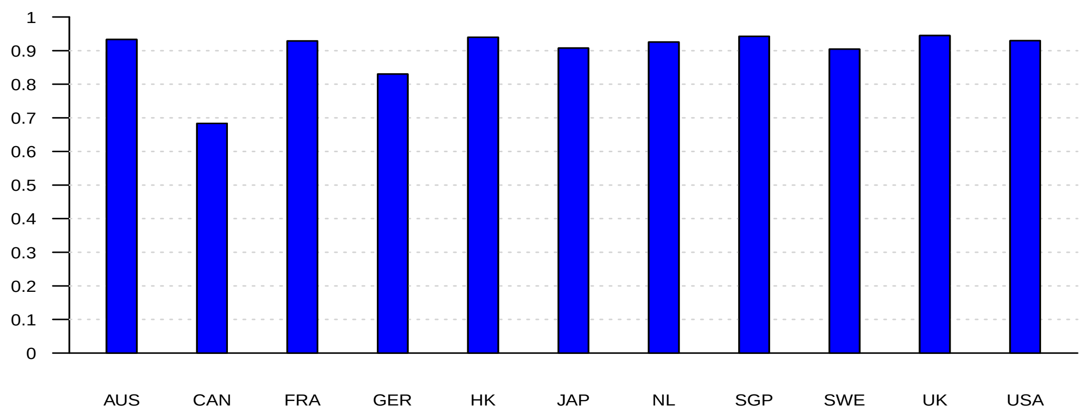


Notes: Notes: The graph shows constant correlations of the GPR Gen Q with the other global indices, depending on the holding period of investors. Monthly return data from 1990:1 to 2014:12 is used.

Appendix G – Comparison of FTSE EPRA/NAREIT- and GPR General Quoted-Subindices

Fig. A and Table A give an impression about the relationship and potential differences between the country indices of the GPR General Quoted and the FTSE EPRA/NAREIT. Most of the correlation coefficients tend to be above 0.9 and deviations in descriptive statistics between countries are rather small. This suggests that empirical tests should yield similiar results and the indices might be seen as substitutes. However, the reader has the possibility to conduct parts of the empirical tests with the FTSE EPRA/NAREIT indices on the accompanying website.

Fig. A:
Constant Correlations between Country Indices



Notes: The graph shows the constant correlations between the EPRA/NAREIT and the GPR General Quoted country indices per country. Monthly total return data from 1991:1 to 2014:12 is used.

Table A:
Descriptive Statistics of GPR General Quoted and FTSE EPRA/NAREIT Country Indices

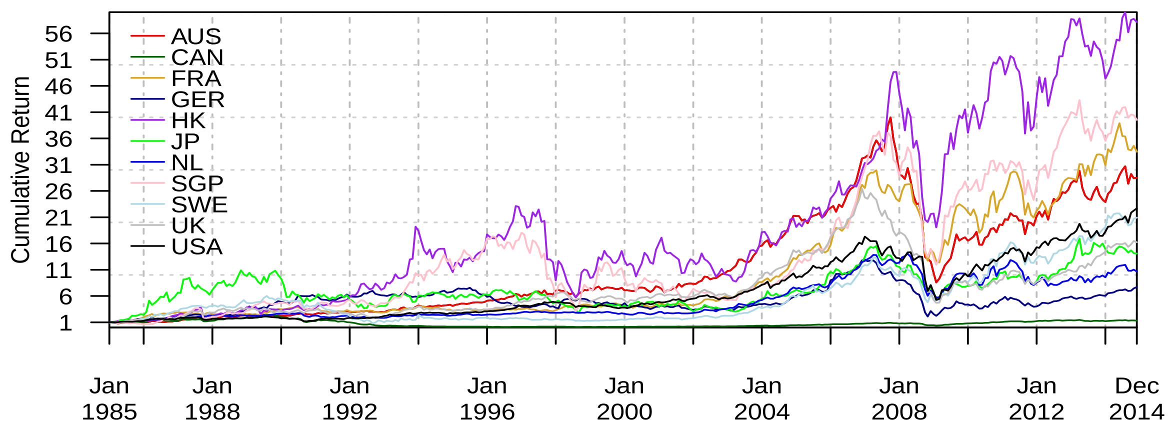
	GPR General Quoted				FTSE EPRA/NAREIT			
	Mean	SD	Sharpe	ES	Mean	SD	Sharpe	ES
Australia	1.031	5.722	0.180	-19.241	0.972	5.970	0.163	-30.727
Canada	0.234	6.551	0.036	-17.712	0.584	6.340	0.092	-28.752
France	0.967	5.852	0.165	-14.885	1.104	6.094	0.181	-15.892
Germany	0.358	7.552	0.047	-8.790	0.578	7.787	0.074	-9.171
Hong Kong	1.477	10.178	0.145	-11.090	1.296	9.928	0.131	-13.577
Japan	0.662	8.142	0.081	-14.450	0.740	9.044	0.082	-15.452
Netherlands	0.709	5.641	0.126	-15.703	0.682	5.642	0.121	-16.613
Singapore	1.352	10.355	0.131	-11.496	1.028	10.686	0.096	-15.432
Sweden	1.018	9.738	0.105	-10.207	0.864	9.701	0.089	-14.354
UK	0.756	6.250	0.121	-16.566	0.670	6.243	0.107	-18.041
USA	1.173	5.624	0.209	-20.219	1.314	5.679	0.231	-19.200

Notes: The table shows descriptive statistics of the EPRA/NAREIT and the GPR General Quoted country indices. The calculations are based on monthly return data from 1991:1 to 2014:12. The ES measure is calculated by means of the Cornish Fisher approximation.

Appendix H – Cumulative Returns and VaR of Country Indices

The appendix offers complementing information for the return enhancement analysis in chapter 5.2.1.

Fig A:
Global – Cumulative Returns of the GPR General Quoted Country Indices



Notes: The Figure depicts the cumulative returns of all country indices over the time period 1985:1-2014:12.

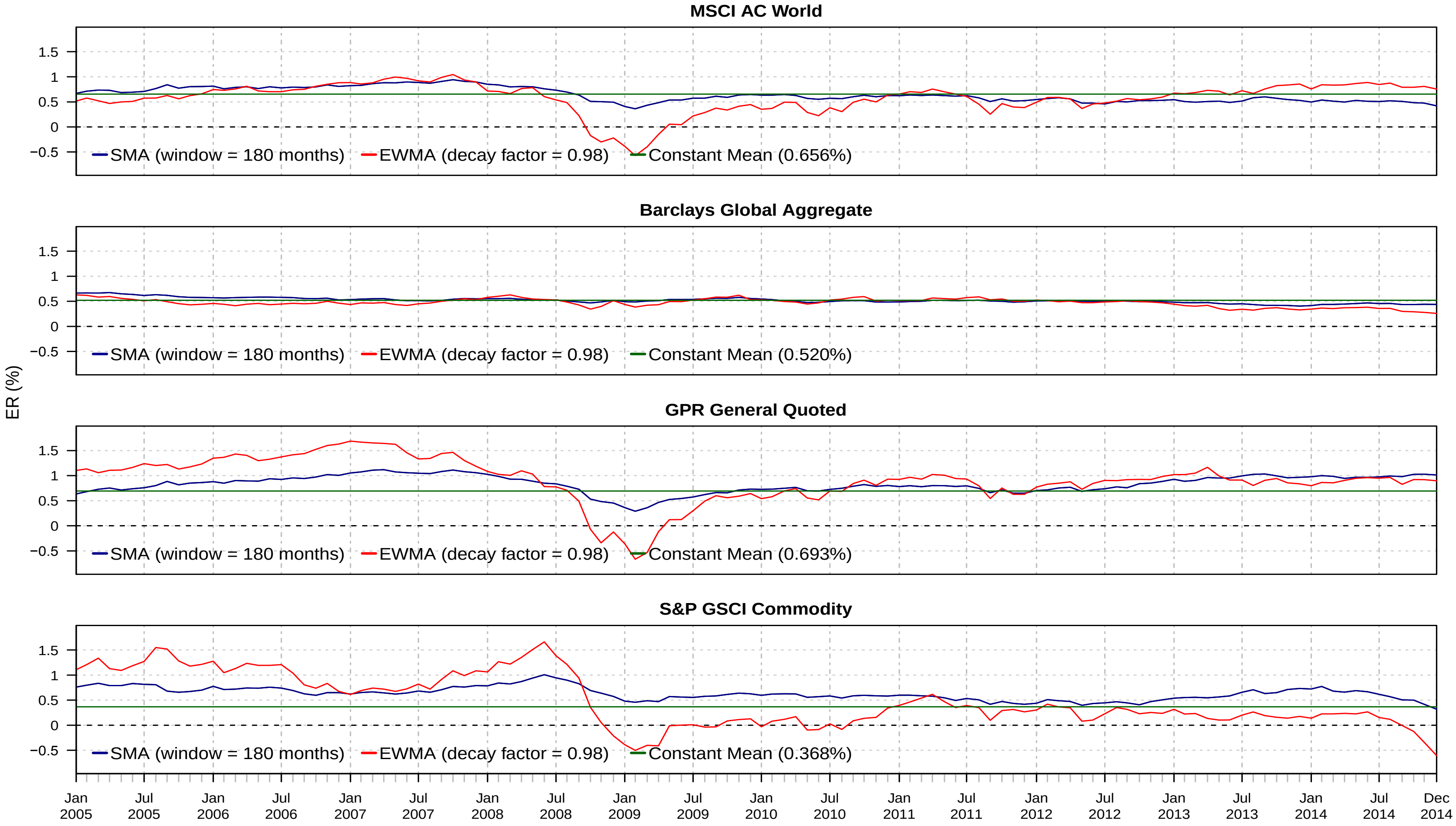
Table A:
Global – VaR per Month

	Real Estate		Equity		Bonds	
	Cornish Fisher	Empirical	Cornish Fisher	Empirical	Cornish Fisher	Empirical
National						
USA	-8.491	-6.723	-6.970	-7.189	-1.636	-1.722
International						
Australia	-9.642	-8.418	-10.950	-9.085	-5.930	-5.733
Canada	-11.308	-10.541	-8.858	-7.471	-3.754	-3.370
France	-8.808	-7.860	-9.537	-10.530	-4.287	-4.423
Germany	-8.543	-8.768	-10.655	-10.525	-4.567	-4.659
Hong Kong	-12.460	-12.115	-11.379	-10.686		
Japan	-9.469	-12.825	-9.005	-8.216	-4.665	-5.097
Netherlands	-8.647	-7.942	-8.997	-9.042	-4.530	-4.751
Singapore	-13.390	-14.454	-11.692	-11.143	-3.059	-2.746
Sweden	-10.182	-12.287	-11.046	-11.281	-4.968	-4.891
UK	-10.173	-8.932	-7.759	-7.247	-4.291	-4.479
Barclays Global Aggregate					-1.318	-1.361
GPR General Quoted	-7.990	-7.510				
GPR 250	-8.679	-7.729				
GPR 250 REITs	-7.628	-6.190				
MSCI World			-7.016	-6.691		
MSCI World All Countries			-7.137	-7.227		

Notes:.If possible the calculations are based on the whole sample (1985:1-2014:12).

Appendix I – Time-varying Means of the global Indices

Fig. A:
Time-varying Means of the global Indices



Notes: The graph depicts the mean return based on different calculation methods. Beside the constant unconditional mean, a SMA with a window of 15-years and an EWMA with a decay factor of 0.98 is displayed. Monthly return data from 1990:1 to 2014:12 is used.

Appendix J – Out-of-sample Tests with Country Indices

The tables below show the descriptive statistics of monthly-rebalanced portfolios with different target SDs. The weighting is based on mean-variance optimization. A fixed rolling window as well as an expanding window is used for the optimization procedues. The expected return equals the mean within the repesctive window and also the covariance matrix solely depends on the returns within the respective window.

Two different portfolio specifications are investigated. First, a global benchmark portfolio that does not have a direct real estate exposure (Table A) and a global benchmark portfolio that already consists of a national real estate investment (Table B). The test portfolios allow for both national and international real estate investments.

Table A:
Global Portfolio **without** an explicit Real Estate Investment + Global Real Estate

		Fixed Rolling Window			Expanding Window		
Target SD	**Portfolio**	**Mean (%)**	**Std. Dev. (%)**	**Ratio**	**Mean (%)**	**Std. Dev. (%)**	**Ratio**
2	Benchmark	0.319	2.232	0.143	0.438	2.041	0.215
	incl. Real Estate	0.414	2.547	0.163	0.472	2.147	0.220
2.5	Benchmark	0.267	2.870	0.093	0.473	2.551	0.185
	incl. Real Estate	0.389	3.314	0.117	0.485	2.747	0.177
3	Benchmark	0.229	3.454	0.066	0.446	3.247	0.137
	incl. Real Estate	0.373	4.000	0.093	0.444	3.431	0.130
3.5	Benchmark	0.195	3.993	0.049	0.407	3.889	0.105
	incl. Real Estate	0.336	4.596	0.073	0.420	4.043	0.104
4	Benchmark	0.141	4.435	0.032	0.467	4.353	0.107
	incl. Real Estate	0.322	4.950	0.065	0.440	4.624	0.095

Notes: An investor who optimizes her portfolio every month based on mean-variance optimization is assumed. The intial window consists of fifteen years. Furthermore, monthly returns from 1985:1 to 2014:12 are used. The allocation is constraint to 60 percent per asset class. Moreover, short-selling is prohibited.

Table B:
Global Portfolio **with** a national Real Estate Investment + Global Real Estate

		Fixed Rolling Window			Expanding Window		
Target SD	**Portfolio**	**Mean (%)**	**Std. Dev. (%)**	**Ratio**	**Mean (%)**	**Std. Dev. (%)**	**Ratio**
2	Benchmark	0.334	2.510	0.133	0.452	2.123	0.213
	incl. int. Real Estate	0.414	2.547	0.163	0.472	2.147	0.220
2.5	Benchmark	0.283	3.201	0.088	0.479	2.706	0.177
	incl. int. Real Estate	0.389	3.314	0.117	0.485	2.747	0.177
3	Benchmark	0.231	3.830	0.060	0.427	3.399	0.126
	incl. int. Real Estate	0.373	4.000	0.093	0.444	3.431	0.130
3.5	Benchmark	0.194	4.372	0.044	0.388	3.923	0.099
	incl. int. Real Estate	0.336	4.596	0.073	0.420	4.043	0.104
4	Benchmark	0.140	4.786	0.029	0.438	4.390	0.100
	incl. int. Real Estate	0.322	4.950	0.065	0.440	4.624	0.095

Notes: Portfolios are rebalanced every month based on mean-variance optimization. The intial window consists of fifteen years. Furthermore, monthly returns of the period 1985:1-2014:12 are used and the allocation is constraint to 60 percent per asset class. Moreover, short-selling is prohibited.

Appendix K – Time-varying Weights of optimized Portfolios (Global)

The graphs below depict the asset class weights based on the portfolio optimization exercises in chapter 5.2.3. Country indices are used and a portfolio manager with a two percent target SD is assumed. Moreover, short selling is forbidden and weights are constrained to 60 percent per asset class and country. The initial window is set to 15 years. Furthermore, the asset class weights equal the sum of the country weights of the respective asset class.
Intriguingly, even after the financial crisis in 2008 the allocation to real estate stays above ten percent but country weights of the real estate portfolio indicate that the allocation to US real estate decreased to zero after the crisis.

Fig. A:
Global –Weights of optimized Portfolios (Fixed Window)

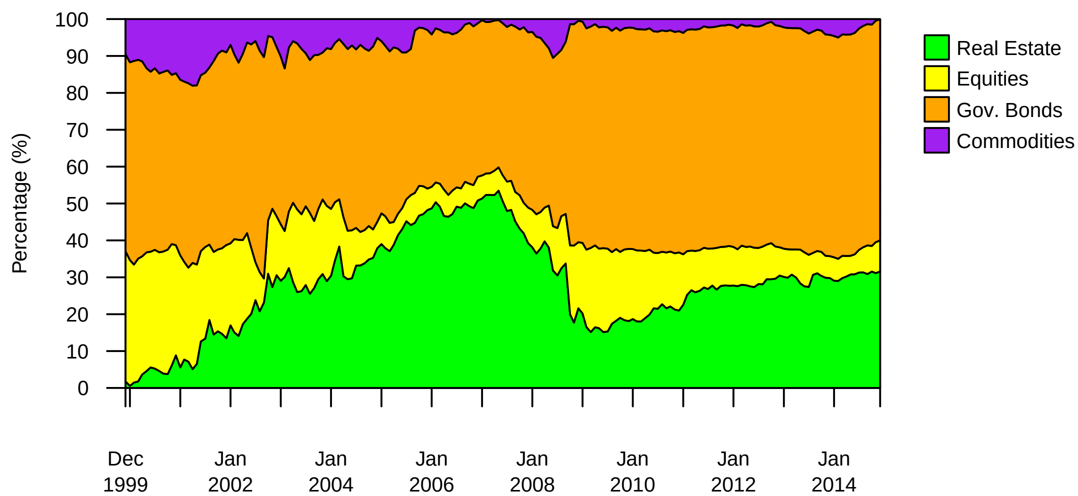
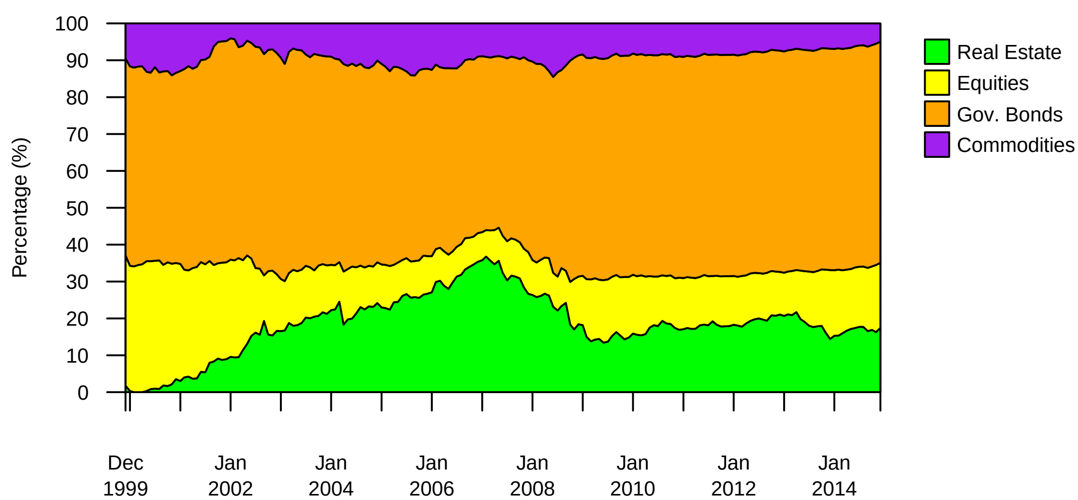


Fig. B:
Global –Weights of optimized Portfolios (Expanding Window)



Appendix L – Summary Tables

The tables below give an overview of the results of the conducted spanning tests, out-of-sample tests and the pros and cons for including listed real estate to a mixed-asset portfolio. The summaries give a good impression of the overall empirical results.

Table A:
Summary of out-of-sample Tests

Benchmark Portfolio	Test Portfolio	Results
National Portfolio without access to an explicit Real Estate Investment	+ National Real Estate	<u>Rolling Window:</u> M Squared of only 7 basis points can be generated and the realized risk is significantly higher than the targeted 2% SD. <u>Expanding Window:</u> Return-Risk Ratio of the portfolio without real estate is better for an investor who targets 2% SD. Only investors, who are willing to take high risks and lower return-risk ratios, might benefit from an investment in terms of a higher M Squared.
Global Portfolio (Country Indices) without access to an explicit Real Estate Investment	+ Global Real Estate (Country Indices)	<u>Rolling Window:</u> M Squared of only 4 basis points for portfolios with a target SD of 2%. Deviation from the target SD is higher for the Portfolio with real estate. M Squared increases up to 22 basis points with higher risk targets but return-risk ratios become considerably lower <u>Expanding Window:</u> M Squared of around 1 basis point considering a target SD of 2%. No strong indications for benefits of higher risk investors.
Global Portfolio (Country Indices) with access to only national Real Estate	+ International Real Estate (Country Indices)	<u>Rolling Window:</u> M Squared of 7 basis points with a target SD of 2%. M Squared increases up 23 basis points with higher targeted risk but the return-risk ratios decrease at the same time. <u>Expanding Window:</u> M Squared of only 1.5 basis points with a target SD of 2%. No strong indications for benefits of higher risk portfolios.

Notes: The Table presents a brief summary of the conducted out-of-samples tests. The portfolios are constrained in form of a maximum allocation of sixty percent per asset class. Furthermore short selling is not allowed and the initial window is set to 15-years.

Table B:
Summary of Spanning Tests

Mixed-Asset Benchmark Portfolio	+ National Real Estate	+ Global Real Estate (Country Indices)	+ Global Real Estate (Broad Indices)
National without Real Estate	No Advantage	Statistically different for SD of two percent	No Advantage
Global without Real Estate (Broad Indices)	Statistically different along the whole Frontier	-	No Advantage
Global without a Real Estate Investment (Country Indices)	No Advantage	No Advantage	No Advantage
Global with a national Real Estate Investment (Country Indices)	-	No Advantage	No Advantage

Notes: The Table presents a summary of spanning tests for different benchmark and test portfolios. The portfolios are constrained in form of a maximum allocation of sixty percent per asset and asset class. Furthermore no short selling is allowed and the optimization is conducted over the whole sample period if possible (1985:1-2014:12).

Table C:
Summary Pros and Cons

Pros	Cons
Return Enhancement (Chapter 5.1.1 & 5.2.1)	
Highest mean returns over the whole sample and over several subperiods suggest absolute return enhancing capabilities for less risk-averse investors.	No indications for superior risk-adjusted returns.
Lower tail risk of many countries compared to the local equity indices.	High volatility and tail risk.
Diversification (Chapter 5.1.2 & 5.2.2)	
Low correlations with other Asset Classes. In particular, on a national level and between country indices.	Decreasing diversification opportunities due to increasing correlations with equity and commodities in times of higher volatility. This indicates tail dependencies with other asset classes.
Less integrated international real estate markets compared to the international equity and bond markets.	
Lower volatility and downside risk of annualized returns for mid-term investors compared to equity and commodities (stronger indications for national indices).	No evidence for mean reversion can be found.
Lower correlations between asset classes for mid-term investors.	
Inflation Hedging (Chapter 5.1.4)	
Positive long-term equilibrium relation with realized and expected inflation. Whereas no for long-term hedging capabilities of equity or commodities are identified.	Rather slow adjustment process towards the equilibrium relation – slower compared to bonds
	In contrast to commodities, no evidence for short-term hedging capabilities can be found.
Portfolio Optimization (Chapter 5.1.3 & 5.2.3)	
Static national as well as global frontiers indicate absolute return enhancement capabilities of a real estate investment for less risk-averse investors.	Most spanning tests cannot find statically significant benefits of adding real estate.
Time-varying M Squared analysis and also the out-of-sample tests suggest that an extension can be beneficial during several time periods.	Out-of-sample-tests do not strongly promote an extension to real estate.
Weights substantiate great additional diversification benefits of real estate – also in low risk portfolios.	

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