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Investment Beliefs of Endowments

Abstract

American university and college endowments now hold close to one-third of their portfolios in private equity and hedge funds. We estimate the implied beliefs of endowments about alternative assets' returns relative to equities and bonds. At the end of 2012, the typical endowment believes that its private equity investments will outperform a portfolio of conventional assets by 3.9% per year and that hedge funds will outperform by 0.7% per year. Out-performance beliefs, particularly for private equity, have increased since 2006. There are significant cross-sectional differences in beliefs and trends: private universities are, on average, less risk averse and have more optimistic beliefs and universities with larger endowments, higher spending rates, and those that rely more on the endowment to meet operational budgets tend to believe that alternatives deliver higher alphas. Taking into account the implied equity exposures in alternative asset positions, the effective equity holding of endowments is approximately 60%.

1 Introduction

In recent years, important institutional investors such as university endowments, sovereign wealth funds, and pension funds have shifted their asset allocation away from standard asset classes like stocks and bonds into alternative investments such as private equity and hedge funds. University endowments in particular have been leaders in the recent trend towards alternative investments. David Swensen's *Pioneering Portfolio Management* articulates the value proposition of this investment style: first, accessing factor returns through non-marketable investments offers an additional liquidity premium to patient investors and second, inefficient asset markets offer astute investors the chance to capture positive alpha by identifying skilled managers. A handful of university endowment officers put these principles into practice in the 1990s and 2000s and were very successful. Many other institutions followed suit (cf. Goetzmann and Oster, 2012).

With the widespread adoption of the alternative investment paradigm, a fundamental concern is whether the experience of the industry first-movers can be successfully imitated. By adopting a new style of investing, are investment managers also expecting to realize future risk-adjusted returns commensurate with the past performance of its most successful practitioners? In this paper, we extract the implicit beliefs held by university endowments about the excess returns they expect to capture by investing in hedge funds and private equity. For hedge funds, we estimate their expected alphas to be somewhat lower than found by industry and academic studies based on historical data. For private equity, we find the expected alphas to be commensurate with, and by some measures, somewhat higher than those found by industry and academic studies, depending on definitions of excess return and the nature of the databases used for analysis.

These beliefs have changed through time. This is interesting because investors in alternative assets face considerable parameter uncertainty. Returns to investments in private equity and hedge funds are not as fully understood as investments in stocks and bonds, for which decades of data are available. Private equity and hedge fund data are less accessible, less reliable, and have been less thoroughly studied by industry and academia. Peer-reviewed academic research on hedge fund and private equity performance is still relatively recent, and improvements in the theory and empirical analysis of these investment strategies are on-going. The change in the beliefs about alternatives over time may reflect the learning process, updating based upon recent performance and reported research. In particular, beliefs about private equity alpha increase dramatically over the seven-year period of our study.

There are also significant cross-sectional differences. Private universities with larger endowments have relatively higher alpha expectations compared to smaller endowments. This may reflect an implicit assumption of increasing risk-adjusted returns to scale; Barber and Wang (2012), for example, document a 3.15% to 3.82% positive alpha earned by Ivy League schools (the early adopters of alternative investing). This range is consistent with the expectations of the average endowment for the alpha generated by their private equity investment alone.

This paper focuses on investors' views about net abnormal returns of alternatives relative to passive equity and bond investments. We assume that asset returns follow a factor model and endowments solve a standard portfolio allocation problem. Using a Bayesian framework, we use information on asset returns and cross-sectional portfolio allocations to estimate the implied views of educational endowment managers about alternatives' net abnormal returns (which we term "alpha"). Our approach allows for updating of beliefs though time, and for risk aversion and alternatives' out-performance beliefs to vary across endowments.

The Bayesian framework allows us to estimate implicit investor beliefs about their capacity to capture excess returns conditional upon both optimistic and pessimistic prior positions with respect to market efficiency, and their past historical performance. Our basic findings are robust to both of these specifications. Investors have high expectations for capturing excess returns through private equity investments—either via manager skill, or an illiquidity premium, or both. We also find that past positive experience influences expectations. This is consistent with the finding of Barber and Wang (2012) who document endowment performance persistence. It can also reflect endowments revising investment beliefs upwards after a period of high returns, which are effects Malmendier and Nagel (2011, 2013) document for individual investors.

Our analysis has implications for the future of university endowments and other institutions strategically committed to large allocations in alternative asset classes. In the fiscal year ending in 2012, the typical endowment expects to earn an alpha of 3.9% in private equity and 0.7% in hedge funds after adjusting these alternatives for their equity and fixed income risk exposures. Elite institutions might continue to earn alpha on their alternative asset portfolios. However, if the expectations of the average institution are overly optimistic, the long-term consequences are at best a growing resource gap between the top and the middle, and at worst a long-term decline in universities' spending power, should illiquidity premiums and alpha in alternative asset classes disappear. Disproportionate beliefs about the performance of financial assets can also distort the saving decision by universities. If institutions are too optimistic about future returns, they will allocate more resources to endowments and forgo internal projects that would have otherwise been undertaken.

This paper is closely related to other studies that back out investors' beliefs from observed asset allocations using a Bayesian framework. Analyzing the holdings of U.S. investors in foreign equities, Pástor (2000) finds that home bias is consistent with investors having a strong prior belief in the domestic CAPM. Similarly, Li (2004) studies the effect of beliefs about the risk of foreign investments on portfolio choices, while Avramov (2004) focuses on the implications for investment decisions of different prior beliefs about stock return predictability. None of these studies investigate endowments or consider private equity or hedge funds. The paper is also related to a large literature on private equity and hedge funds, which have focused primarily on historical performance and which we later review. These papers have not examined investor out-performance beliefs in these asset classes using actual holdings.

This paper is organized as follows. Section 2 reviews current evidence regarding excess returns to alternative investments. Section 3 presents an asset allocation model where alternative assets deliver out-performance compared to standard equity and bond factors. Section 4 describes the endowment data. Section 5 contains the main results of the paper and estimates investment beliefs of endowments with various priors. Section 6 concludes.

2 Background

Because the rationale for investing in hedge funds and private equity is in part the potential for delivering positive alpha in excess of a passive benchmark, it is important to examine the current and past evidence on hedge fund and private equity manager skill. The research literature, as well as industry studies were part of the information set of endowment managers and thus likely affected their priors and their revision of beliefs about asset allocation parameters in general and beliefs about the excess return to alternatives in particular. Given the lack of reliable, long-term return information about both hedge funds and private equity, past and current empirical studies by leading researchers are important sources of information for portfolio managers about the prospects of alternative investments.

2.1 Hedge Funds

Evidence on abnormal returns earned by hedge funds is mixed. The early academic evidence reports positive abnormal returns for the industry. Fung and Hsieh (1997) documents considerable non-linearity in hedge fund returns with respect to standard asset pricing factors and introduced additional controls. They find that hedge funds over the period of their study were a good investment. Ackermann, McEnally, and Ravenscraft (1999) find that hedge funds outperformed mutual funds over the period 1988 through 1995, but do not, on average, provide positive risk adjusted returns. In contrast, Brown, Goetzmann, and Ibbotson (1999) find evidence of positive risk-adjusted performance in a database of off-shore hedge funds over the same time period. Presumably these and related studies that followed influenced institutional investor expectations about the potential for positive alpha.

Subsequent studies modified these early results to some extent. Bailey, Li, and Zhang (2004) document the outperformance of hedge funds under the null of no arbitrage, even when non-linear factor payoffs are considered. Kosowski, Naik, and Teo (2007) examine the risk adjusted performance of hedge funds over the period 1990 to 2002 using fairly sophisticated measures. Their results concur that hedge funds over this extended period appear to have delivered positive performance persistent at the annual horizon. More recently, Ibbotson, Chen, and Zhu (2011) find that hedge funds delivered an average alpha of 3% per year over 1995 to 2009. A recent update of the study (unpublished) by the authors through 2012 lowers this to about 2.5% per year due to lower industry returns since 2009. Dichev and Yu (2011) use a dollar-weighted measure to show that alphas realized by hedge fund investors were much lower than those derived from time-weighted rates of return due to the timing of investment flows. Fung and Hsieh (2000) argue that fund of fund returns are a more appropriate basis for evaluating returns realized by investors in hedge funds. Alphas derived from the University of Massachusetts CISDM fund of hedge fund indices are lower at 2.2%, but still higher than the implied beliefs we estimate from the endowment database.

In addition to aggregate studies of the hedge fund industry, a number of researchers have examined conditional strategies for accessing manager outperformance. Avramov et al. (2011) for example, show that interacting macroeconomic conditions with manager selection yields positive results. Some studies of manager persistence support the potential for benefiting from "hot hands" in the hedge fund industry. Capocci and Hübner (2004), Kosowski, Naik, and Teo (2007), and Fung et al. (2008) show that even though there appears to be some short-run persistence, only a small group of hedge funds is able to generate alpha over longer horizons (one to three years). On the other hand, Jagannathan, Malakhov, and Novikov (2010) report significance persistence in hedge fund returns.

Some papers questioned or re-interpreted the historical evidence of positive hedge fund alphas. Griffin and Xu (2009) find little evidence of differential or superior trading skill by hedge funds during the tech bubble. Malkiel and Saha (2005) argue that survivorship bias and backfill bias loom large in any reliance on historical hedge fund data and on this basis question whether prior empirical evidence is reliable enough for forming expectations of future performance. Aiken, Clifford, and Ellis (2013) point out that the voluntary nature of hedge fund reporting to commercial databases means that the worst performers are not represented and thus the severity of the lower till of hedge fund returns is biased upwards.[1]

[1] The issue of survival bias in the databases may cut two ways, however. Linnainmaa (2013) estimates

In sum, hedge fund researchers have documented positive risk adjusted returns within the hedge fund universe using imperfect but commercially available databases. These excess returns have declined somewhat in recent years and scholars caution that data issues may be biasing the evidence. Research on the effect of selective reporting, survivorship and backfilling has yet to yield a comprehensive approach to proper adjustment of expectations. Studies of relative performance and performance persistence support beliefs in the ability to select superior managers from the population, although the survival effects on persistence studies are also unclear.

For the purposes of our analysis, we presume that the academic studies cited above, in addition to related studies from academia and practice were inputs to the formation of investor expectations with respect to hedge funds, and that priors of positive alphas would have been consistent with the reported empirical evidence over the period 1997 through 2011. Our estimate of the implied beliefs about hedge fund alphas is 0.7% per year, which is below most of the empirical studies cited. This low estimate may reflect distrust in the empirical evidence about hedge fund alphas, a skepticism about the future potential for generating alpha, or doubt about the ability of the average fund to access high alpha managers.

2.2 Private Equity

We find that expectations about private equity alpha at the end of fiscal year 2012 are 3.9%. This roughly corresponds to the illiquidity premium estimated by Franzoni, Nowak, and Phalippou (2012), the private equity return differential over the S&P 500 reported by Harris, Jenkinson, and Kaplan (2012) and the differences in geometric means between the widely used industry benchmark Cambridge Associates US Private Equity Index and the S&P 500. It is higher than estimates of alpha delivered by private equity investments for which alpha is defined as the residual component of return not explained by exposure to a multi-factor equity benchmark that includes small cap, value, and liquidity factors.[2]

that mutual fund performance is downward biased by certain measures from fund closings due to negative exogenous shocks as opposed to poor skill. Presumably the same holds true for hedge funds as well.

[2] Franzoni, Nowak, and Phalippou (2012) for example, show that private equity alphas net of Fama-French and Pastor-Stambaugh factors is roughly 0.4% per annum. Harris, Jenkinson, and Kaplan (2012) correct for buyout funds' exposure to small cap and value factor and estimates of net alpha in the

Estimating abnormal returns to private equity investment is empirically challenging because of the lack of time-series market-based valuation. This problem was pointed out clearly by Gompers and Lerner (1997). Traditional measures used to evaluate private equity, such as the internal rate of return do not lend themselves to adjustment for systematic risk exposures. Techniques such as comparison to public market equivalents are used in lieu of time-series data but provide only approximate estimates of the capacity for private equity investment to outperform an equivalent investment in marketable securities. Given these limitations to empirical analysis and the lack of a comprehensive dataset, it is not surprising that academic studies on private equity have reported mixed evidence on private equity alphas.

Some of the earliest academic works relevant to the formation of expectations by managers in the dataset we study are Moskowitz and Vissing-Jørgensen (2002), who, using entrepreneurial returns as a proxy, find that private equity investment underperforms, and Kaplan and Schoar (2005) who find no evidence of outperformance over the S&P 500 net of fees. Harris, Jenkinson, and Kaplan (2012) on the other hand, use a recently available proprietary database and find significant outperformance for U.S. private equity funds net of an S&P 500 benchmark. Higson and Stucke (2012) employ another large dataset of funds with vintage years from 1980 to 2008 and find that private equity outperforms the S&P500 by more than 5% per year. Axelson, Sorensen, and Stromberg (2013) find that leveraged buyout deals outperform the market by approximately 8.5% per year, gross-of-fees. They also document an increase in the level of alpha over their sample period, finding an outperformance of more than 14% per year over the period 2001 to 2007. Robinson and Sensoy (2011) study the sensitivity of performance evaluations to the value of beta used for risk-adjustment. They find that for betas close to one, the level of alpha is rather insensitive and estimate an over performance of about 12% per year.

Some studies have focused on the relative contribution of private equity investments in excess of standard tradable equity factors. Private equity buyout funds typically purchase

neighborhood of 1.5% to 2.5% per annum—about half the premium estimates using only the S&P 500 as a benchmark. Phalippou (2013) estimates the factor-exposure-adjusted annual premium for buyout funds to be zero or lower using a micro-cap benchmark appropriate for buyout funds acquiring smaller companies.

small undervalued companies and use leverage to do so. Thus it is logical to benchmark them against measures of similar publicly tradable companies. Edwards and Caglayan (2001), Phalippou and Gottschalg (2009), and Franzoni, Nowak, and Phalippou (2012) argue that after correctly accounting for leverage, illiquidity, size, and value, investments in private equity do not, on average, generate positive alphas. Harris, Jenkinson, and Kaplan (2012) note that similar adjustments reduce the size of the premium by about half.

As with the hedge fund literature, cross-sectional evidence is also relevant for investors' expectation formation about their ability to access positive alpha. Lerner, Schoar, and Wongsunwai (2007) note that universities have been relatively successful at selecting private equity investments. Other studies have found evidence of performance persistence consistent with the presumption that access to top managers may deliver consistently higher returns.

In sum, recent evidence supports expectations of a positive private equity in excess of the S&P 500, while research also suggests that some or all of this premium is compensation for exposure to equity factors such as leverage, size, value and liquidity. However, all of these studies are limited by the nature of the data. We would expect endowment managers to be aware of these results and adjust their expectations—and level of parameter uncertainty—in light of them.

2.3 Endowment Performance

Endowment income constitutes a significant, and growing fraction, of universities' operating budgets. Brown et al. (2014) document that universities practice "endowment hoarding" as they seek to preserve the value of the fund following a negative shock. Consequently, past performance impacts universities' operations and aspirations. They also find that, contrary to conventional wisdom, endowments adjust their spending rules often. Moreover, these changes in payouts are more likely to follow low past returns and low outflows. Almost half of the endowments in the sample adjust the rule at least once, while a quarter adjust spending rates every year.

There is a small but growing empirical literature on university endowment performance. Lerner, Schoar, and Wang (2008) study the same database we use and find that the largest endowments and endowments of the most elite academic institutions outperformed—and these were also the group that relied most on alternative investments. Brown, Garlappi, and Tiu (2010) use the same data to study whether endowments added value through allocation timing decisions. They find some evidence of skill which they believe to be under-utilized. Barber and Wang (2012) find no evidence that university endowments on average added alpha through timing or manager or security selection, although they find no signs of negative risk-adjusted returns. Even though there is some evidence that endowments outperform other institutional investors within private equity investments, a later study by Sensoy, Wang, and Weisbach (2013) documents that the superior performance disappears in the late period of their sample. They argue that the performance gap can be explained by endowments' better access to top venture capital partnerships when the industry was less mature.

3 Model

Investors can cheaply invest in passively managed equity and fixed income funds. Alternative asset classes, which include private equity and hedge funds, are actively managed at a much higher cost. Even if equities and fixed income are also actively managed, the fees on alternative investments are usually much higher than fees on traditional equity and bond products. Taking this as a starting point, we develop an asset allocation model based on Treynor and Black (1973) which takes equities and fixed income as factors. Alternative assets are exposed to factor risk, and they may exhibit alpha—out-performance that cannot be attributed to the equity and fixed income factor exposures, but which is generated with additional idiosyncratic risk. The methodology accommodates prior views, which may be held by investors or specified by the econometrician, on the risk premiums of alternative assets.

3.1 Factors and Alternative Assets

We extend the Treynor and Black (1973) model to multiple factors and assets with non-zero risk-adjusted returns. We assume there are N_f tradable factors whose excess returns, f, follow

$$f = \mu_f + \varepsilon_f, \tag{1}$$

where μ_f is a size N_f column vector of expected excess returns and ε_f is a vector of independent and identically distributed (iid) normal shocks with covariance matrix Σ_f. The covariance matrix need not be diagonal, but must be full rank. We take U.S. equities, foreign equities, and U.S. bonds as factors in our empirical work.

There are N_a alternative assets whose excess returns, r_a, follow the data-generating process

$$r_a = \alpha + \beta f + \varepsilon_a \tag{2}$$

In our empirical work, we take private equity and hedge funds as alternative assets. We capture the co-movement of these alternative asset classes with equity and bond factors through the factor loadings β, which is an $N_a \times N_f$ matrix. Idiosyncratic shocks, ε_a, are assumed to be iid normal with covariance matrix Σ_a. We assume a factor model structure, so the idiosyncratic shocks are orthogonal to the factor shocks, $\varepsilon_f \perp \varepsilon_a$. However, idiosyncratic shocks may have non-zero cross-correlations.

The alternative assets exhibit abnormal returns, α, which is an $N_a \times 1$ column vector. Alpha is the mean return that alternative assets have in excess of their factor exposures. It can reflect mispricing or the fact that our set of N_f factors is incomplete. Either interpretation is consistent with endowments holding alternatives to seek returns which cannot be generated by holding plain-vanilla equities and bonds.[3] It is important to note

[3] We assume that both μ_f and α are constant, and therefore, investment opportunities do not change over time. This is done for tractability, but time-varying expected returns can be incorporated in this framework, generating an additional hedging demand from investors. We work in a mean-variance asset allocation context following Pástor and Stambaugh (1999, 2000), Pástor (2000), Avramov (2002), Avramov and Zhou (2010), and many others. We do allow estimates of alphas and risk aversion to change over time in our empirical work, as we detail below.

that in our model alternative asset returns are assumed to be normally distributed. If the actual return distribution exhibits fat tails and skewness, our estimate of alpha is a lower bound since investors would require a higher alpha to compensate for these extra sources of risk.

It is worth commenting on our selection of equities and bonds as factors. A voluminous literature has used other systematic factors including value-growth, size, momentum and other factors in equities following Fama and French (1993), and factors incorporating option payoffs to model hedge funds (cf. Fung and Hsieh (2001)). All of these factors involve dynamic trading, and cannot be done at effectively zero cost. In contrast, the passive equity and bond market-capitalization index returns we use are investable by any investor at negligible cost. Arguably, except for the largest and most sophisticated endowments, exposure to some of the more sophisticated factors involving significant leverage, high turnover rebalancing, and complex, data-intensive investment processes can only be obtained by investing in hedge funds or private equity.

3.2 Portfolio Allocation

Investors maximize a mean-variance utility function with risk aversion γ:

$$\max_{\pi} E(r_p) - \frac{\gamma}{2}\sigma_p^2, \tag{3}$$

where $E(r_p)$ and σ_p^2 are the expected return and variance of the portfolio, respectively. The weight in risky assets, π, which has a dimension $N_f + N_a$, can be partitioned into holdings on factor securities and alternative assets, $\pi = [\pi_f^\top, \pi_a^\top]^\top$. The remaining weight in the risk-free asset, with return r_f, ensures the portfolio weights sum to one.

The portfolio's expected excess return, $\mu_p = E(r_p) - r_f$, is given by

$$\begin{aligned} \mu_p &= \pi_f^\top \mu_f + \pi_a^\top (\beta\mu_f + \alpha) \\ &= \tilde{\pi}_f^\top \mu_f + \pi_a^\top \alpha, \end{aligned} \tag{4}$$

where

$$\tilde{\pi}_f = \pi_f + \beta^\top \pi_a. \tag{5}$$

We can interpret $\tilde{\pi}_f$ as the total implicit portfolio weight on factors since alternative assets co-move with the factors. We examine this implicit factor exposure in our empirical work for different optimal portfolios.

Since shocks to the excess return of the portfolio can be written as

$$
\begin{aligned}
(r_p - r_f) - \mu_p &= \pi_f^\top \varepsilon_f + \pi_a^\top \left(\beta \varepsilon_f + \varepsilon_a\right) \\
&= \tilde{\pi}_f^\top \varepsilon_f + \pi_a^\top \varepsilon_a, \qquad (6)
\end{aligned}
$$

the variance of portfolio returns is equal to

$$
\sigma_p^2 = \tilde{\pi}_f^\top \Sigma_f \tilde{\pi}_f + \pi_a^\top \Sigma_a \pi_a. \qquad (7)
$$

The optimal portfolio allocations that maximize mean-variance utility in equation (3) are then given by

$$
\pi_f^* = \frac{1}{\gamma} \left(\Sigma_f^{-1} \mu_f - \beta \Sigma_a^{-1} \alpha\right) \qquad (8)
$$

$$
\pi_a^* = \frac{1}{\gamma} \Sigma_a^{-1} \alpha. \qquad (9)
$$

The optimal factor holdings in equation (8) can be broken into two terms. The first is the standard static demand for factor securities, $\frac{1}{\gamma}\Sigma_f^{-1}\mu_f$, when no alternative assets are available. The second term, $\frac{1}{\gamma}\beta\Sigma_a^{-1}\alpha$, adjusts the benchmark allocations by taking into account the factor exposures of alternative assets. Equation (9) shows that the investor holds alternative assets only if they have non-zero alpha.

Combining the previous expressions, we can express the risk aversion coefficient, γ, as

$$
\gamma = \frac{\sigma_p^2}{\mu_p} = \frac{\tilde{\pi}_f^{*\top} \Sigma_f \tilde{\pi}_f^* + \pi_a^{*\top} \Sigma_a \pi_a^*}{\tilde{\pi}_f^{*\top} \mu_f + \pi_a^{*\top} \alpha}. \qquad (10)
$$

Thus, portfolio holdings in the data can be used to estimate endowments' risk aversion.

3.3 Endowment Beliefs

It is natural to estimate the implied investment beliefs of endowments in a Bayesian framework. We use the model in Section 3.2 and infer endowments' beliefs given observed asset allocations and historical returns, following Pástor and Stambaugh (1999, 2000), Avramov (2002), Avramov and Zhou (2010) and others. Our approach is similar in that we treat some assets as factors (U.S. equity, international equity, and bonds), and model the alternative assets (private equity and hedge funds) as active returns with alpha. However, previous studies only use the time-series of returns to conduct statistical inference about alphas. In our approach, we use both past returns and actual portfolio holdings to infer investors' beliefs.

Denoting the return history and portfolio holdings as $\mathbf{X}$, we estimate the distribution of alternative assets' alphas given the observed data and a prior belief. To illustrate the approach, consider the case where we only estimate the parameter α. We construct the posterior distribution

$$p(\alpha|\mathbf{X}) \propto p(\mathbf{X}|\alpha)p(\alpha). \tag{11}$$

To construct the likelihood function, $p(\mathbf{X}|\alpha)$, we assume that the portfolio weights π_f and π_a in the data are equal to the weights in equations (8) and (9), respectively, plus some observation error:

$$\pi_f = \pi_f^* + u_1 \tag{12}$$

$$\pi_a = \pi_a^* + u_2, \tag{13}$$

where u_1 and u_2 are iid normal random variables with diagonal covariance matrices $\Sigma_{\pi f}$ and $\Sigma_{\pi a}$, respectively. The errors u_1 and u_2 are orthogonal to each other, and are orthogonal to the factor shocks, ε_f, and the shocks to the alternative assets, ε_a.

We assume several prior beliefs, $p(\alpha)$. When a uninformative, or flat, prior is used, alpha is estimated from data on returns and portfolio holdings alone. We also use informative priors: a pessimistic prior which assumes that alternative assets have a negative return of -4% per year and an optimistic prior with a return of 4% per year, each with a standard deviation of 2%.

The estimated posterior distribution, $p(\alpha|\mathbf{X})$, can be used in several ways. First, the posterior mean, $E(\alpha|\mathbf{X})$, can be interpreted as the implied investment belief that the typical endowment possesses in order to justify its portfolio holdings in alternative assets. The posterior distribution can also be used to compute other moments and confidence intervals. This gives a picture of the dispersion of endowments' beliefs and also can be used to judge statistical significance. Finally, by computing the posterior distribution of alpha for various prior beliefs, we can gauge how robust the investment views of endowments are.

In our empirical work, we estimate the posterior distribution of all parameters, not just α. The full set of parameters is $\Theta = \{\mu_f, \Sigma_f, \alpha, \beta, \Sigma_a, \Sigma_{\pi,f}, \Sigma_{\pi,a}, \gamma\}$. We use flat priors for all parameters except α and μ_f. We motivate the informative priors for μ_f as follows. Our sample for factor returns is longer than the sample we use for alternatives. Since mean-variance portfolio weights are sensitive to the mean parameters, we parameterize the prior distribution of the factor excess returns, μ_f, in a way that allows us to change the weight given to the return data vs. the asset allocation data.[4] In particular, we assume a prior density centered on the time-series mean and a scale parameter proportional to the time-series covariance matrix. The parameter ν controls the informativeness of the prior distribution, so that higher values of ν increase the weight given to factor returns. For example, if $\nu = T_f/(T_f + T_i)$, where T_f is the length of the factor sample and T_i is the length of the data on asset holdings, the prior distribution is flat and the posterior of μ_f is proportional to the likelihood. If $\nu = 1$ the posterior distribution is degenerate at the historical average excess return, so only time-series information is used to estimate μ_f. We consider the uninformative prior as our baseline specification, but we estimate the model with other values for ν for robustness.

We estimate the model using a Bayesian Markov Chain Monte Carlo (MCMC) approach. The estimation procedure generates posterior distributions of the parameters by iteratively drawing from conditional densities which take into account all the information contained in assets' time-series returns, the cross-section of investors' allocations, and prior distributions. For a detailed exposition of the estimation algorithm, please see the

[4] See Best and Grauer (1991) and Green and Hollifield (1992).

appendix.

3.4 Endowment Heterogeneity

In the mean-variance model of Section 3.2, portfolio weights are determined by investor risk aversion and assumptions on the data-generating process of returns. Risk aversion, however, varies across endowments, and different endowments are also likely to have different beliefs on the alpha of alternative assets. To capture this heterogeneity, we assume that risk aversion, γ, and the alpha belief, α, depend on endowment size, past returns, spending rules, and other characteristics. Denoting these observable characteristics as Z, we assume that the risk aversion and alpha for endowment i are given by, respectively,

$$\gamma_i = \gamma_0 + \gamma_1 Z_i \tag{14}$$

$$\alpha_i = \alpha_0 + \alpha_1 Z_i, \tag{15}$$

where Z_i is a vector of endowment i's characteristics, γ_0 and α_0 are constants, and γ_1 and α_1 are vectors which allow endowments' risk aversion to linearly depend on the characteristics.

We standardize the set of endowment characteristics $Z = \{Z_i\}$ so that it is mean zero and unit variance at any point in time. Thus, the parameters γ_0 and α_0 represent the average level of risk aversion and the average view on the magnitude of alternative assets' abnormal returns, respectively. This also allows us to interpret the γ_1 and α_1 coefficients as representing the effect of a one-standard deviation change across the cross section of endowment characteristics. We assume that endowments agree on parameters other than γ and α.

In addition, we allow α_0 to vary over time. We can plot a time series of α_0 and examine the evolution of endowments' beliefs. In fixing the other parameters for the full sample, we assume that time-series changes in average allocations to alternative assets are mainly driven by changing views on α_0. This is reasonable, since we have a relatively long time series of factor returns, and estimates of covariance parameters contain much less sampling error than estimates of means (see, for example, Merton, 1980). There is also some time-series variation in γ and alphas that result from changing endowment characteristics.

4 Data

4.1 Asset Allocation of Endowments

The portfolio allocation of most college and university endowments in the United States are voluntarily reported to the National Association of College and University Business Officers (NACUBO) and Commonfund. We use the NACUBO/Commonfund survey for the years 2006 to 2012. The database contains approximately 800 public and private university endowments which are surveyed on a yearly basis. In addition to asset allocations, we also have general information about universities: their size, spending rates, and past endowment returns. Universities report numbers to NACUBO and Commonfund for their fiscal year ends, which for most universities is June 30.

NACUBO uses ten asset categories, which are listed in the left-hand column of Table 1. To obtain a more parsimonious group of asset classes, we form five groups: U.S. stocks, fixed income, foreign stocks, private equity, and hedge funds. We group private equity, real estate and venture capital into a "private equity" class, and the "hedge fund" category includes energy and natural resources, commodities, managed futures, marketable alternative strategies, and distressed debt. We treat cash as a risk-free asset. Endowments are not restricted from using leverage; Harvard University, for example, had a -5% cash holding in 2008 and a -3% holding in 2009.[5] The majority of endowments, however, do not use short positions.

Using just five asset classes has several advantages. First, it minimizes the effects of parameter sensitivity to data errors and mitigates well-known problems of extreme portfolio positions resulting from estimating large number of parameters. Second, the classification of assets differs from endowment to endowment, so a hedge fund investing in distressed commercial mortgage assets might be defined as a "marketable alternative strategy" for one endowment, a "distressed debt" fund for another, or even as a "private

[5] See "Liquidating Harvard," Columbia CaseWorks #100312.

equities real estate" fund. Grouping assets minimizes these reporting biases. Third, using fewer asset class groups is consistent with our aim to estimate broad investment views of endowments as a whole.

Table 2 reports the average allocations in the five asset groups. Over the sample period, there is a strong trend towards divesting from domestic stocks and increasing holdings in alternative investments. In 2006, the average allocation to U.S. stocks was 46% while at the end of 2012 that value was only 32%. At the same time, the average share of funds allocated to private equity increased from 5% in 2006 to 9% at the end of 2012. The corresponding average allocation to hedge funds increased from 12% to 19%. This is shown clearly in Figure 1, which plots the average allocation to U.S. equities and to alternatives defined as the sum of the average allocation to private equity and hedge funds. Average alternative asset holdings rose from 17% in 2006 to 28% in 2012.

Table 2 shows that there is significant cross-sectional dispersion in the allocation to U.S. equities and alternatives. We address this in our model in two ways. First, some heterogeneity in portfolio weights is captured by the endowment-specific observation error we specify around the model-implied weights (equations (12) and (13)). We also capture heterogeneity by allowing endowment risk aversion and beliefs about alternative asset class alphas to depend on university-specific characteristics (equations (14) and (15)). Panel A also shows that in contrast to the decreasing holdings of U.S. equities and the increasing weights on alternatives, the weights on fixed income and foreign stocks have stayed relatively constant in our sample. In addition, these asset classes also have lower cross-sectional standard deviations.

4.2 Endowment Characteristics

In Table 3, we report summary statistics for various endowment characteristics: type of institution (public or private), size of the endowment in millions of U.S. dollars, percentage of the fund that is spent each year, percentage of the university's budget that is funded by the endowment, and the performance of the endowment over the past year. Table 3 lists the number of observations available every year, and the number of non-missing values. In the estimation, we do not restrict ourselves to using only endowments for which all variables are observed; our algorithm is able to use the full sample and infer values for missing observations (see the appendix).

In our sample, approximately 60% of the endowments fund private colleges or universities. The average fund size over the sample period is $463 million. There are large differences in size both across time and across endowments. The average size reaches its peak in 2008 before the financial crisis, shrinks by 35% during 2009, and recovers during 2012 to $518 million. The smallest 10% of endowments have less than $13 million over our sample period, and the largest 10% manage more than $847 million. The largest endowment in our sample, well-known from other sources to be Harvard University, has assets totaling approximately $31 billion as of 2012.

Endowment income plays a very important role in meeting operational budgets for universities. The average spending rate from endowment funds is 4.4%, and this is very persistent over time. There is modest variation in the spending rate across universities. The share of the university budget funded by the endowment exhibits more cross-sectional variation, with the typical university relying on the endowment to meet around 10% of their operations. Finally, endowment performance has significantly varied across universities. This may reflect the different experiences of endowments in alternative investments, or their different abilities to market time.[6]

4.3 Asset Class Returns

For each asset class we choose a well-known index with two key characteristics. First, we focus on indices with a long history of returns. Time-series information is relevant for identification since it pins down the moments of the distribution of returns. Second, we require indices to be marketable in order to avoid the problems associated with appraisal-based pricing, which induces artificial smoothing. All our return data are at the monthly frequency. Since implied beliefs of endowments may be sensitive to the estimates from the shorter samples of the alternative asset returns, we examine robustness with various

[6] Lerner, Schoar, and Wang (2008) document considerable heterogeneity in endowment returns, some of which is due to their different holdings in equities and alternative assets. Brown, Garlappi, and Tiu (2010) show that a significant fraction of cross-sectional differences in endowment performance comes from the (lack of) ability to market time asset classes.

priors in our empirical work.

We use the S&P 500 index, the Ibbotson Associates Long-Term Government index, and the MSCI World ex-U.S.A index as proxies for domestic stocks, fixed income, and foreign stocks, respectively. Our data samples are January 1926 to December 2012 for domestic stocks and bonds, while the sample for international stocks starts in January 1973. As a proxy for alternative investments, we use the HFRI Fund of Funds index and the S&P Listed Private Equity index. In these cases, monthly returns are available starting from January 1990 and January 1994, respectively. One possible issue is that our proxy for hedge fund is not publicly traded and therefore may underestimate the true volatility of the asset class. This can potentially bias downward our estimate for alpha. We partially address this issue in Section 5.4 by using different proxies. Finally, we use the Ibbotson Associates 30-day T-Bill returns as a risk-free rate to construct excess returns.

Table 4 reports summary statistics for excess returns on the asset classes. Domestic and international stocks have the highest average excess returns, at 5.91% and 4.20%, respectively. They also exhibit similar levels of volatility and have Sharpe ratios of 0.31 and 0.24, respectively. Fixed income has a lower Sharpe ratio of 0.25.

Private equity has the lowest Sharpe ratio of 0.14 among the asset classes. This Sharpe ratio is significantly lower than the performance of private equity typically reported in academic studies, such as Robinson and Sensoy (2011) and Harris, Jenkinson, and Kaplan (2012). This is because we use a listed equity index for private equity, rather than an index representing direct, illiquid private equity investment. Infrequent trading, the use of appraisals, and selection bias where we tend to observe market valuations only when the underlying valuations are high, all potentially cause illiquid, direct private equity indices to substantially under-state their true volatility (see Ang and Sorensen (2012), for a summary). The volatility of private equity during the 1994-2012 period is 24.4%, which is above the stock market volatility of 19.0% over the 1926-2012 period—which is expected since private equity funds typically hold non-diversified portfolios with high idiosyncratic volatility (cf. Ewens et al., 2013).

Hedge funds have a Sharpe ratio of 0.65, which is driven by the unusually low volatility of aggregate returns, at only 5.8%, in our sample. Hedge fund abnormal performance has declined over time, as Dichev and Yu (2011) and others note. Hedge funds have a lower correlation with equities, at 0.54, than private equity, which has a correlation with equities of 0.67; since private equity is a form of equity, it is not surprising that unlisted equity is highly correlated with standard listed equity.

5 Results

In Section 5.1, we report estimates of the model and the implied beliefs about the risk and return of investments. We track endowments' implied beliefs about alphas over time. Section 5.2 addresses the heterogeneity across endowments. In Section 5.3, we present results with informative priors, and priors which put different weights on the time-series of returns vs. the cross section of asset holdings. We conduct a series of robustness checks in Section 5.4.

5.1 Investment Beliefs

The estimated parameters are shown on Table 5. In solving the mean-variance model (equation (3)), we assume a risk-free rate of 3.5%. The amount of risk-free holdings by endowments is small, at less than 5% (see Table 2), and so the results are insensitive to the choice of the risk-free rate. Panel A reports the estimates of the factor, factor loading, and observation error parameters of the model. Compared to historical time-series data, our model generates significantly larger expected excess returns for domestic stocks and international equities. Estimated excess returns for U.S. stocks is 15.6% per year and for foreign stocks is 12.9% per year. On the contrary, fixed income excess returns, at 2.8% per year, are very similar to historical performance. This result is interesting, given that publicly traded securities remain a substantial proportion of the average endowment portfolio despite the move toward alternatives. These results imply that endowment portfolio managers are not implicitly expecting lower equity or bond risk premiums, at least with respect to historical averages, despite reducing the weights on these asset classes to fund the increased allocations to alternatives.

In Panel B of Table 5, we detail endowments' risk aversion levels. The average level of risk aversion for an endowment over the whole sample is 7.48. We find that private endowments are significantly more risk tolerant. Our estimates also show that larger endowments are more risk tolerant, however the coefficient is not statistically significant. Additionally, the spending rate is positively correlated with risk aversion, while the percentage of the university budget financed by the endowment and its past return are negatively related to γ. These latter effects are likewise not statistically significant.

Panel C reports the average level of alpha beliefs for private equity and hedge funds. For both asset classes, alpha beliefs are positive—and statistically and economically significant each year. We also find evidence of heterogeneity due to endowment characteristics. Beliefs about both private equity and hedge fund alphas are higher for private endowments, funds with more assets under management, endowments with higher spending rates (only significantly so for hedge funds), and endowments that fund a higher proportion of universities' operating budgets. For hedge funds, positive past year returns are significantly associated with positive alpha beliefs, however this is not true for private equity. Given that past year returns on private equity investments are not reliably observable by investors, this difference is not surprising.

5.1.1 Time-Varying Investment Beliefs

Figure 2 shows how the average view on the level of mispricing has evolved over time. For both alternative asset classes our alpha estimates increase over the sample, reflecting the observed trend in endowment allocations into alternative investments. The average view on private equity alpha increases from 1.39% per year in 2006 to 3.89% per year in 2012. Interestingly, our model generates an alpha that is larger than the OLS estimate from historical returns (0.54% per year) and larger than the alphas reported in the academic literature on the subject available prior to, and concurrent with, most of the period of the study. Although our proxy for private equity performance is imperfect as a measure of long-term performance, and empirical studies are limited by available private equity data, this suggests the prevalence of fairly aggressive positive beliefs about private equity. Thus, despite limited and imperfect available information, endowments believe private equity has significant alpha.[7]

Beliefs about hedge funds' abnormal returns also rise over time. In 2006 the average view is 0.29% per year while in 2012 it increases to 0.66% per year. In contrast to the private equity case, the historical alpha of 1.53% per year is higher than the average belief by university endowments. In Figure 2, there are no significant effects of the Great Recession on endowments' beliefs and allocations. One could argue that our results are biased by the fact that endowments were forced to hold on their private equity holdings due to illiquidity. In that case, observed allocations may not have fully reflected endowments' beliefs on returns of alternative investments. However, not only we find no decrease in estimated alphas after the liquidity crisis subsided, our results show that expectation of abnormal returns actually increased. This is consistent with Brown et al. (2014), who show that payouts are an important margin of adjustment after financial shocks: in particular, they document that endowments *reduce* payouts after negative market shocks. Endowments' beliefs may have increased to self-justify their higher spending requirements and inability to rebalance.

An alternative way of viewing endowments' investment beliefs is to consider Sharpe ratios and information ratios, which we report in Table 6. Consistent with our results on endowments' beliefs on alphas, implied Sharpe ratios and information ratios on alternatives increase over our sample period. The increase in information ratios is particularly steep. Over 2006 to 2012, the average posterior mean of the information ratio increased from 0.059 to 0.164 for private equity and from 0.0561 to 0.129 for hedge funds. Sharpe ratios and information ratios for hedge funds and private equity are quite similar, unlike the much smaller implied alphas for hedge funds in Panel C of Table 5. The low alpha beliefs are partly driven by the low volatility of the fund-of-hedge fund returns, and once expected returns are normalized by volatility in the Sharpe ratio and information ratio measures, there is not much difference in the private equity and hedge fund beliefs.

[7] A survey of institutional investors by Dhar and Goetzmann (2006) taken just prior to the sample period found that that managers were relatively less comfortable in basing their expectations about future performance on past returns to hedge funds and private equity, compared to traditional asset classes.

5.1.2 Dispersion of Investment Beliefs

Figure 3 shows the cross-sectional distributions of the risk aversion coefficient and the views on private equity and hedge fund alphas. The two-humped shape of the distribution of risk aversion in the top panel is due to the large fixed effect related to the type of institution variable (public or private). Since this is a dummy variable it generates a bimodal distribution. Although Panel A of Table 5 documents that private universities have significantly lower risk aversions, the economic difference is not large. A striking result is the large difference in belief dispersion between private equity and hedge funds in the middle and bottom panels. The model is able to capture a large degree of heterogeneity in alpha for private equity. This is reflected by the large α_1 coefficient associated with the size characteristic. An endowment that is one standard deviation larger than the average fund expects a higher out-performance of 2.56% per year. This may reflect the fact that endowments have truly less disagreement on the level of mispricing of hedge funds, or that our model is not able to capture this cross-sectional dispersion because it is associated with missing or unobservable characteristics.

An interesting exercise is to compute the certainty equivalent (CE) returns across endowments; what riskless return would endowments demand to give up their current risky portfolios? The higher the expected future returns for both factor securities and alternative investments, the higher the certainty equivalents. Figure 4, Panel A graphs the distribution of CEs in our sample where the unit of observation is endowment-year. The mean and median of the distribution are 8.8% and 8.7% respectively. The bottom endowment-year decile would demand a risk-free rate of 8.4% while for the top decile the required rate of return is 9.3%. Panel B shows the first and second moments of the cross-sectional distribution over time. The average CE return increases over our sample period, consistent with endowments requiring higher compensation to give up risky asset holdings due to more optimistic views on future performance of alternative investments.

In summary, the results indicate that the average endowment believes that there are significant gains from holding private equity investments. The expected level of alpha is significantly higher than our historical proxy and the empirical evidence found in the literature. There are two possible explanations for this. First, endowments may think that they have superior selection skills and are able to pick the managers that generate the highest alpha. This selection skill is not reflected on historical measures of performance. Second, they may expect higher conditional risk premia on liquidity or other risk factors in the future.

5.2 Endowment Holdings and Factor Exposures

In this section we address the extent to which we can explain the observed cross-sectional differences in endowment allocations. To do so, we use our estimated parameters to compute model-implied optimal asset allocation for each endowment. The results are shown in Table 7. The model accurately fits the observed average allocations to the different asset classes. Our results, however, do not account for the full cross-sectional variation observed in the data. Cross-sectional standard deviations of portfolio weights implied by the model are much smaller than in the data, especially for fixed income and foreign stocks. Model-implied average allocations also do not change over time as much as observed weights. Both results can be explained, to some extent, by the assumptions made in the model. In particular, we assume that endowments differ only in their risk aversion coefficient and their view on alpha. Allowing for changes in the average view on factors' excess returns could improve the fit of the model. The literature has found other factors that matter, which we do not observe. Goetzmann and Oster (2012), for example, find that endowment decisions to change asset allocation is conditional upon strategic considerations, including rivals' performance. Gilbert and Hrdlicka (2013) argue that a number of other unobserved university fundamentals may explain relative level of investment in risky assets, including the marginal productivity of internal projects, the influence of self-interested stakeholders, and binding constraints on payout rates.

Given the factor structure of our model, we can also derive the implicit allocations to factor securities. We use the estimated factor loadings, β, and the observed allocations to compute $\tilde{\pi}_f$ for each endowment in our sample. Figure 5 shows non-parametric kernel densities for the cross-sectional distribution of allocations to the three factor securities. Endowments' exposure to U.S. stocks and fixed income are larger than the explicit weight on those asset classes. In the case of international equities, the distributions of implicit and explicit weights are similar.

The difference between the actual and the effective allocations to equity is pronounced. The average fund has close to 62% of its wealth allocated to stock-like securities after controlling for the equity factor exposure in private equity and hedge funds. Endowments not viewing their total factor exposure may be significantly under-estimating their exposure to equity risk—as many universities found out in 2008 and 2009.

5.3 Informative Priors

The results in previous sections assumed uninformative priors for the average level of mispricing α_0 and factor excess returns μ_f. In this section, we impose informative priors and investigate how posterior distributions and optimal allocations change.

First, we assume that an investor, before observing the asset dynamics and endowments allocations, has optimistic or pessimistic prior beliefs on alternative investments. In the optimistic case, we assume that the investor's prior on the level of abnormal returns for both private equity and hedge funds are normally distributed with an annualized mean of 4% and a standard deviation of 2%. In the pessimistic case, we specify a normal prior with a mean of −4% and a standard deviation of 2%.

Figure 6 graphs the average of the posterior distribution of alpha for both optimistic and pessimistic priors. The effect of the prior appears to be asymmetric. Under the optimistic prior, the posterior distribution for private equity shifts up by almost 1% per year. The estimated α_0 of the baseline specification is lower than the 5th percentile of the new distribution. When the prior is pessimistic, the estimated α_0 shifts down by less than 0.5% per year. The same behavior is observed for hedge funds. In sum, observed returns and allocations provide the investor with a sufficient amount of information to significantly update his prior belief on alternatives' alpha. Even with an informative negative prior, endowments exhibit beliefs with positive abnormal returns on private equity and hedge funds.

Second, we consider different prior distributions for the factor securities' excess returns μ_f since, as we show above, estimated mean excess returns in the baseline specification are significantly higher than historical values. Therefore, we examine the effect of increasing the weight given to historical returns in our estimation procedure. Table 8 shows the estimated average view on alpha for private equity and hedge funds. When ν is equal to $T_f/(T_f + T_i) = 0.19$ the prior is uninformative and we are in the baseline case. As ν increases, the scale parameter of the prior decreases and more weight is given to time-series information. In all cases the model is able to fit average allocations reasonably well. Table 8 shows that investment beliefs about private equity and hedge fund alphas decrease as ν increases. The intuition of this result is as follows. A more informative prior pushes μ_f down to its historical average. In order to fit observed allocations to factor securities, the level of risk aversion has to go down as well. But a lower risk aversion increases the weights on mispriced securities unless the average level of alpha also decreases. Nevertheless, our general conclusions hold. The view on the level of alternatives' alpha increases over the sample. Furthermore, the lower the weight on historical returns, the larger the gap between estimated alphas for private equity and hedge funds.

5.4 Robustness

We check robustness of our results by re-estimating the model under different assumptions and specifications. We consider the following cases:

1. Historical return data starting only from 1970.

2. Restricting the sample to the ten largest university endowments: Brown, Columbia, Cornell, Darthmouth, Harvard, Princeton, University of Pennsylvania, Yale, Stanford, and MIT. Many of these large endowments were first-movers into alternative assets.

3. Winsorization of funds' observed characteristics at the 5th and 95th percentiles.

4. Different proxy for private equity, the Cambridge Associates Private Equity Index. Since this index is only available on a quarterly basis starting from 1990, we convert all monthly returns to quarterly returns in this exercise.

5. Different proxy for hedge funds, the Dow Jones Credit Suisse Hedge Funds Index, which is available starting from 1994.

6. Dropping all endowments with no holdings in hedge funds or private equity from the sample.

7. Collapsing private equity and hedge funds into a single asset class. As a proxy for this broad alternative asset class, we use the weighted average of the baseline indexes for private equity and hedge funds. The cross-sectional average of the shares invested in each subclass are used as weights (approximately 30% private equity and 70% hedge funds).

Table 9 shows the estimated risk aversion, γ, and the average alternatives' alpha, α_0. In all specifications, the view on the level of mispricing for both private equity and hedge funds is significantly greater than zero and increases over time. Also, as in our main specification, the view on private equity's alpha is significantly larger than the one for hedge funds with one exception: when we use the Cambridge Associates Private Equity index, both mispricing levels are similar. Another interesting result is that large endowments appear to have more aggressive views with the estimated α_0 being approximately twice as large. Winsorizing endowments' characteristics has almost zero effect. We also obtain similar results when both asset classes are grouped into a single alternative investment. The average view on alpha is significantly greater than zero and increases over time reaching 1.3% in 2012. In sum, endowments expect alternatives' alpha to be high, perhaps because they have demonstrated past capability in capturing alpha in the past and expect to continue to do so in the future.

6 Conclusion

Colleges and universities rely to varying extent upon endowment income to support their missions. The asset allocation policy figures heavily in endowment management, and this, in turn depends upon expectations about risk, return, correlations, and liquidity of various asset classes. There has been a pronounced recent trend towards alternative investments, despite the fact that the risk-return trade-offs of these asset classes are the least well understood. Inference about alternative asset classes, particularly illiquid ones is hampered by imperfect quality and quantity of information. For both private equity and hedge funds, researchers over the past decade have sought to improve the understanding of these investments through empirical studies. This research may well have played a role in endowment priors over the period of our study. Larger holdings of alternative assets suggests that endowments are accepting higher levels of uncertainty in exchange for the potential of high expected returns.

In this paper we address this tradeoff by modeling the allocation decision in a Bayesian framework. We use observed asset allocations to private equity and hedge funds, together with data on alternatives' past returns and standard equity and bond factors, to compute expectations on alternatives' alpha. We find that the typical endowment expects these asset classes to generate significant returns in excess of a benchmark adjusted for systematic risk exposure. There are, however, significant differences in expectations across private equity and hedge funds. The average endowment expects private equity to outperform hedge funds on a risk-adjusted basis. Our results may be due to the average endowment extrapolating the success of the largest and most successful university endowments who moved early into alternative assets. It may be due to a preference for relative uncertainty. The results are even more puzzling in light of the high expectation for the plain-vanilla equity risk premium. Further work on the expectation-formation mechanism is warranted.

Estimation Appendix

A Gibbs Sampler

Denote $X = (f, r_a, \pi_f, \pi_a)$ the set of all time-series and cross-sectional information. The objective is to estimate the parameters' joint posterior distribution given observed factor returns, alternative strategies returns, endowments' asset allocations and their observable characteristics, Z:

$$p(\Theta|X, Z),$$

where $\Theta = \{\mu_f, \Sigma_f, \alpha_0, \alpha_1, \beta, \Sigma_a, \Sigma_{\pi,f}, \Sigma_{\pi,a}, \gamma_0, \gamma_1\}$ is the set of all parameters. We denote Θ_- as the set of parameters less the parameter of interest. We use a Gibbs sampler to sample from the joint posterior by specifying conditional distributions for each parameter: $p(\mu_f|\Theta_-, X, Z)$, ..., $p(\gamma|\Theta_-, X, Z)$. The posterior conditional distributions can be found using Bayes' theorem as the product of the likelihood function and the parameter's prior.

$$\begin{aligned} p(\mu_f|\Theta_-, X, Z) &\propto p(\pi_f, f, r_a|\Theta, Z)p(\mu_f^0) \\ &\vdots \\ p(\gamma|\Theta_-, X, Z) &\propto p(\pi_f, \pi_a, f, r_a|\Theta, Z)p(\gamma^0). \end{aligned}$$

For μ_f and the measurement error variances, Σ_π, $\Sigma_{\pi f}$, and $\Sigma_{\pi a}$, the posterior distributions can be derived in closed form. However, it is not possible to get analytical expressions for the remaining parameters, so we use the Random Walk Metropolis-Hasting algorithm. Given some initial parameters values θ_0, we assume that candidate draws for the nth iteration follow a multivariate random walk,

$$\hat{\theta}_{n+1} = \theta_n + \sigma_\theta w_{n+1},$$

where w_{n+1} is a standardized normal, and σ_θ is a scaling parameter.

The new draw is accepted with probability

$$\min\left\{\frac{p(X|\hat{\theta}_{n+1}, \Theta, Z)q(\theta_n)}{p(X|\theta_n, \Theta)q(\hat{\theta}_{n+1}, Z)}, 1\right\},$$

where $q(\cdot)$ is the prior distribution density. If the draw is accepted, then $\theta_{n+1} = \hat{\theta}_{n+1}$, otherwise $\theta_{n+1} = \theta_n$. Note that since $\hat{\theta}_{n+1}$ follows a random walk, $q(\hat{\theta}_{n+1}) = q(\theta_n)$ and the last term from the numerator and the denominator cancel out.

We run the MCMC algorithm for one million iterations. The first 800,000 draws are used to calibrate the diffusion coefficients σ_θ and are discarded at the end. During this calibration stage if the Metropolis-Hastings' acceptance ratios are below 5%, diffusion parameters are reduced in half. Similarly, if acceptance ratios are above 50% the variance is doubled. The posterior distribution is then computed using the last 200,000 draws. Convergence is fast.

B Factors' Expected Excess Returns, μ_f.

Assume a multivariate normal prior $N(\mu_0, \Sigma_0)$, where μ_0 is an N_f column vector and Σ_0 is a $N_f \times N_f$ matrix. Since both the likelihood and the prior are conditionally normal, we obtain an analytical expression for the posterior by completing the square.

$$\begin{aligned} p(\mu_f|\Theta_-, X, Z) &\propto p(\pi_f, f|\Theta)p(\mu_f) \\ &\propto p(\pi_f|\Theta)p(f|\Theta)p(\mu_f) \\ &\propto N(\mu_f|\mu_n, \sigma_n), \end{aligned}$$

where the second line follows from the independence assumption between factor dynamics and allocation measurement errors. The mean and variance of the posterior distribution are

$$\begin{aligned} \Sigma_n &= \left(\Sigma_0^{-1} + T_f\Sigma_f^{-1} + \sum_i \tilde{\Sigma}_{\pi,f}^{-1}\right)^{-1} \\ \mu_n &= \Sigma_n \left(\Sigma_0^{-1}\mu_0 + T_f\Sigma_f^{-1}\bar{f} + \sum_i \tilde{\Sigma}_{\pi,f}^{-1}\tilde{\pi}_f\right), \end{aligned} \tag{A-1}$$

where

$$
\begin{aligned}
\tilde{\pi}_f &= \gamma \Sigma_f \pi_f + \Sigma_f \beta \Sigma_a^{-1} \alpha_a \\
\tilde{\Sigma}_{\pi,f}^{-1} &= \frac{1}{\gamma^2} \left(\Sigma_f \Sigma_{\pi,f} \Sigma_f^{\top} \right)^{-1}.
\end{aligned}
$$

Both expressions in equation (A-1) are intuitive. The posterior variance is the inverse of the sum of the likelihood and prior inverse variances, weighted by the number of observations. The posterior mean is the average of the likelihood and prior means weighted by the information matrix and the number of observations. The first term is a function of the prior distribution parameters, while the second and third terms are functions of observed excess return means and average allocations. The parameter Σ_0 characterizes the informativeness of the prior. If it is large, the prior is diffuse and the posterior mean does not depend much on the prior. If it is small, the posterior mean remains close to the prior. More observations and lower variance of factor returns, increase the weight of the historical mean on the value of μ_n. The same intuition applies for the information provided by asset allocations.

Assume that $\Sigma_0 = \Sigma_f / T_0$ and $\mu_0 = \bar{f}$. We can rewrite equation (A-1) as

$$
\begin{aligned}
\Sigma_n &= \left(\nu \Sigma_f^{-1} + (1-\nu) \Sigma^{-1} \right)^{-1} \frac{1}{\tilde{T}_f + T_i} \\
\mu_n &= \Sigma_n (\tilde{T}_f + T_i) \left(\nu \Sigma_f^{-1} \bar{f} + (1-\nu) \Pi_f \right),
\end{aligned}
$$

where

$$
\begin{aligned}
\nu &= \frac{\tilde{T}_f}{\tilde{T}_f + T_i} \\
\tilde{T}_f &= T_0 + T_f
\end{aligned}
$$

and

$$\Sigma^{-1} = \frac{1}{T_i} \sum_i \tilde{\Sigma}_{\pi,f}^{-1}$$
$$\Pi_f = \frac{1}{T_i} \sum_i \tilde{\Sigma}_{\pi,f}^{-1} \tilde{\pi}_f.$$

If $T_0 = 0$, the prior is uninformative and the posterior is unchanged. If $T_0 \to \infty$, then $\nu = 1$ and we have a degenerate posterior distribution at the time-series mean. As we increase T_0, the weight given to the time-series information increases.

C Error Variance

Assume an inverse Wishart prior $IW(\nu_0, \Psi_0)$. By Bayes' theorem,

$$\begin{aligned} p(\Sigma_u|\Theta_-, X, Z) &\propto p(\pi|\Theta)p(\Sigma_u) \\ &\propto IW(\nu_N, \Psi_N), \end{aligned}$$

where

$$\begin{aligned} \nu_N &= \nu_0 + T_\pi \\ \Psi_N &= \Psi_0 + \sum_{t=1}^{T_\pi} (\pi_t - \pi^*(\Theta))(\pi_t - \pi^*(\Theta))^\top. \end{aligned}$$

Since measurement errors for factor allocations and mispriced securities allocations are independent, the same procedure can be used for drawing $\Sigma_{\pi f}$ and $\Sigma_{\pi a}$.

D Factor Covariance, Σ_f

By Bayes' theorem the posterior of Σ_f is given by

$$p(\Sigma_f|\Theta_-, X, Z) \propto p(\pi_f|\Theta, Z)p(f, |\Theta)p(\Sigma_f).$$

We use Metropolis-Hastings to draw from the posterior. We specify a candidate draw $\hat{\sigma}_f$, which is accepted with probability

$$\min\left\{\frac{p(\pi_f|\hat{\sigma}_f,\Theta_-,Z)p(f|\hat{\sigma}_f,\Theta_-)}{p(\pi_f|\Sigma_f,\Theta_-,Z)p(f|\Sigma_f,\Theta_-)},1\right\}.$$

E Alternatives' Alpha Parameters, α_0 and α_1

By Bayes' theorem we have

$$p(\alpha_0,\alpha_1|\Theta_-,X,Z)\propto p(\pi_f|\Theta,Z)p(\pi_a|\Theta,Z)p(f,r_a|\Theta)p(\alpha_0,\alpha_1).$$

We use Metropolis-Hastings with an acceptance probability of

$$\min\left\{\frac{p(\pi_f|\hat{\alpha}_a,\Theta_-,Z)p(\pi_a|\hat{\alpha}_a,\Theta_-,Z)p(f,r_a|\hat{\alpha}_A,\Theta_-)}{p(\pi_f|\alpha,\Theta_-,Z)p(\pi_a|\alpha,\Theta_-,Z)p(f,r_a|\alpha,\Theta_-)},1\right\}.$$

F Risk Aversion Parameters, γ_0 and γ_1

By Bayes' theorem we have

$$p(\gamma_0,\gamma_1|\Theta_-,X,Z)\propto p(\pi_f|\Theta,Z)p(\pi_a|\Theta,Z)p(\gamma_0,\gamma_1).$$

Metropolis-Hastings algorithm is used with an acceptance probability of

$$\min\left\{\frac{p(\pi_f|\hat{\alpha}_a,\Theta_-,Z)p(\pi_a|\hat{\alpha}_a,\Theta_-,Z)}{p(\pi_f|\alpha,\Theta_-,Z)p(\pi_a|\alpha,\Theta_-,Z)},1\right\}.$$

G Factor Loadings, β.

By Bayes' theorem,

$$p(\beta|\Theta_-,X,Z)\propto p(\pi_f|\Theta,Z)p(f,r_a|\Theta)p(\beta).$$

The acceptance probability of the Metropolis-Hastings step is equal to

$$\min\left\{\frac{p(\pi_f|\hat{\beta},\Theta_-,Z)p(f,r_a|\hat{\beta},\Theta_-)}{p(\pi_f|\beta,\Theta_-,Z)p(f,r_a|\beta,\Theta_-)},1\right\}.$$

H Alternatives Covariance, Σ_a.

By Bayes' theorem, the posterior of Σ_a is given by

$$p(\Sigma_a|\Theta_-, X, Z) \propto p(\pi_f|\Theta, Z)p(\pi_a|\Theta, Z)p(f, r_a, |\Theta)p(\Sigma_a).$$

We use Metropolis-Hastings with acceptance probability

$$\min\left\{\frac{p(\pi_f|\hat{\sigma}_a, \Theta_-, Z)p(\pi_a|\hat{\sigma}_a, \Theta_-, Z)p(f, r_a|\hat{\sigma}_a, \Theta_-)}{p(\pi_f|\Sigma_a, \Theta_-, Z)p(\pi_a|\Sigma_a, \Theta_-, Z)p(f, r_a|\Sigma_a, \Theta_-)}, 1\right\}.$$

I Inferring Missing Endowment Characteristics

We assume that endowments' observable characteristics, Z, can be modeled as a multivariate normal random variables with mean μ_Z and covariance matrix Σ_Z. When some fund-year pair characteristics are missing, we group the known variables into Z_1 and the unobserved ones in the vector Z_2:

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = N\left(\begin{bmatrix} \mu_{Z1} \\ \mu_{Z2} \end{bmatrix}, \begin{bmatrix} \Sigma_{Z,11} & \Sigma_{Z,12} \\ \Sigma_{Z,21} & \Sigma_{Z,22} \end{bmatrix}\right).$$

We can compute the conditional mean and the conditional covariance matrix of Z_2 given the observed information Z_1 as

$$\hat{\mu}_{Z2} = \mu_{Z2} + \Sigma_{Z,21}\Sigma_{Z,11}^{-1}(Z_1 - \mu_{Z1})$$
$$\hat{\Sigma}_{Z,22} = \Sigma_{Z,22} - \Sigma_{Z,21}\Sigma_{Z,11}^{-1}\Sigma_{Z,12}$$

Given μ_{Z2} and $\Sigma_{Z,22}$, we draw new values for the unobserved characteristics in each iteration of the Gibbs sampler for the missing Z_2:

$$\hat{Z}_2 \sim N\left(\hat{\mu}_{Z2}, \hat{\Sigma}_{Z22}\right).$$

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Table 1: Asset Allocations of Endowments

The table lists the asset allocation categories in the NACUBO-Commonfund study in the left-hand column, and our classification in the right-hand column.

NACUBO Category	Group
Cash	Cash
U.S. stocks	U.S. stocks
Fixed income	Fixed income
Foreign stocks	Foreign stocks
Private equities real estate Venture capital Private equity	Private equity
Energy and natural resources Commodities managed futures Marketable alternative strategies Distressed debt	Hedge funds

Table 2: Endowments' Asset Allocation

The sample consists of university endowments in the NACUBO-Commonfund Study of Endowments from 2006 through 2012. We list cross-sectional means and standard deviations (in parentheses) of endowments' allocations to domestic stocks, fixed income, international stocks, private equity, and hedge funds using the groupings of the original NACUBO assets in Table 1. Allocations are in percent.

Year	2006	2007	2008	2009	2010	2011	2012	Average
Cash	4.34	5.16	4.14	7.46	6.02	5.70	5.37	5.63
	(8.49)	(11.49)	(7.93)	(11.31)	(9.01)	(9.17)	(8.26)	(9.56)
U.S. stocks	45.55	42.37	38.07	33.58	32.44	32.51	31.95	35.74
	(16.94)	(16.84)	(17.72)	(16.25)	(16.46)	(16.39)	(15.83)	(17.22)
Fixed income	20.00	17.79	18.98	21.55	21.78	18.99	19.84	20.03
	(11.43)	(9.26)	(10.4)	(11.51)	(11.71)	(10.56)	(11.35)	(11.09)
Foreign stocks	13.49	15.76	15.03	14.29	14.75	16.42	15.25	15.03
	(9.19)	(9.66)	(9.13)	(8.44)	(8.05)	(8.26)	(8.03)	(8.62)
Private equity	5.06	5.80	8.03	7.28	7.25	8.06	8.79	7.31
	(6.08)	(6.67)	(8.38)	(8.78)	(8.89)	(9.29)	(9.95)	(8.66)
Hedge funds	12.08	13.12	15.73	15.85	17.75	18.29	18.74	16.31
	(12.96)	(12.93)	(14.47)	(14.84)	(15.52)	(14.92)	(14.78)	(14.69)

Table 3: Endowments' Characteristics

The sample consists of university endowments in the NACUBO-Commonfund Study of Endowments from 2006 through 2012. The table reports summary statistics for the following characteristics: the number of private vs. public universities, assets under management (in million of dollars), the spending rate, the percentage of the fund that is spent each year, the percentage of the University's budget funded by the endowment, and the funds' performance during the previous year. We report the cross-sectional mean and standard deviation each year, along with various percentiles of the cross-sectional distribution. We also report the number of non-missing observations, N.

		2006	2007	2008	2009	2010	2011	2012	Average
Private	Count	298	351	295	502	507	489	483	2,925
	N	448	534	453	791	799	768	766	4,559
Size	Mean	437	518	529	343	422	511	518	463
	10%	9	12	13	12	13	17	14	13
	50%	75	104	117	68	73	94	92	84
	90%	683	1,003	1,065	665	779	914	933	847
	σ	2047	1762	1814	1166	1615	1933	1934	1748
	N	448	534	453	791	799	768	766	4,559
Spending Rate	Mean	4.62	4.47	4.40	4.37	4.53	4.57	4.15	4.43
	10%	3.10	3.00	3.20	2.27	1.85	2.56	2.47	2.58
	50%	4.54	4.50	4.40	4.50	4.90	4.70	4.25	4.50
	90%	6.00	5.80	5.75	5.75	6.38	5.99	5.40	5.98
	σ	1.77	1.63	1.47	1.88	1.89	3.32	1.52	2.09
	N	437	517	441	758	771	753	747	4,424
% Budget	Mean	.	8.85	9.94	13.41	10.59	9.30	8.63	10.31
	10%	.	0.20	0.00	0.00	0.00	0.00	0.00	0.00
	50%	.	4.31	4.35	4.70	3.25	3.15	3.01	3.80
	90%	.	22.00	26.30	41.84	30.00	24.12	23.00	29.00
	σ	.	13.00	16.71	21.08	18.60	16.39	15.40	17.57
	N	0	344	358	721	718	676	672	3,489
Past Return	Mean	10.54	17.33	-2.69	-18.76	11.95	19.26	-0.33	4.86
	10%	6.87	13.40	-7.13	-24.00	8.40	14.42	-3.20	-18.30
	50%	10.22	17.50	-2.85	-19.10	12.20	19.81	-0.50	8.80
	90%	14.70	20.87	2.14	-12.90	15.40	23.50	2.39	20.10
	σ	3.57	3.38	3.78	5.26	3.23	4.31	2.67	13.63
	N	418	512	437	748	769	740	750	4,374

Table 4: Asset Class Excess Returns

The table shows annualized averages, standard deviations, and correlations for excess returns on the following asset classes: domestic stocks, fixed income, international equities, private equity, and hedge funds. The statistics are computed from monthly returns. U.S. equities are proxied by the S&P 500 from 1926 through 2012. Fixed income is represented by the Ibbotson U.S. Long-Term Government Bond Index for the same period. For international stocks, we use the MSCI International World ex-U.S. Index from 1970 through 2012. For private equity and hedge funds we use Standard & Poors' Listed Private Equity Index, starting in 1994, and the HFRI Fund of Funds Composite Index, starting in 1990. Correlations are computed using the longest available common data sample between each variable.

	U.S. Stocks	Fixed Income	Foreign Stocks	Private Equity	Hedge Funds
Period	1926 - 2012	1926 - 2012	1970 - 2012	1994 - 2012	1990 - 2012
Mean	0.0591	0.0206	0.0420	0.0338	0.0375
Volatility	0.1903	0.0823	0.1750	0.2442	0.0575
Sharpe Ratio	0.31	0.25	0.24	0.14	0.65
Correlations					
U.S. Stocks	1.00	0.09	0.66	0.73	0.54
Fixed Income		1.00	0.05	-0.27	-0.11
Foreign Stocks			1.00	0.72	0.56
Private Equity				1.00	0.67
Hedge Funds					1.00

Table 5: Parameter Estimates

The table lists parameters of the model estimated using asset returns from 1926 to 2012 and endowment allocations from 2006 to 2012. We use uninformative priors for all parameters. Both the view on the alpha of alternative investments and the risk aversion coefficients are assumed to be linear functions of funds' observable characteristics (see equation (14)): whether the college is private, endowment size, spending rate, the proportion of the budget met by endowment revenue, and the return over the past year. The characteristics are cross-sectionally normalized at each point in time. Parameter estimates are annualized. We report posterior means and standard deviations (in parentheses).

Panel A: Factor, Factor Loading, and Observation Error Parameters

		U.S. Stocks	Fixed Income	Foreign Stocks	Private Equity	Hedge Funds
Factors						
μ_f		0.1558	0.0283	0.1287		
		(0.0163)	(0.0030)	(0.0135)		
Σ_f	U.S. Stocks	0.0347	0.0034	0.0254		
		(0.0016)	(0.0002)	(0.0011)		
	Fixed Income		0.0072	0.0020		
			(0.0003)	(0.0001)		
	Foreign Stocks			0.0360		
				(0.0013)		
Alternative Assets						
β	Private Equity	1.5642	0.6873	-0.4657		
		(0.0787)	(0.0635)	(0.0542)		
	Hedge Funds	0.1802	0.1127	0.0361		
		(0.0269)	(0.0251)	(0.0227)		
Σ_a	Private Equity				0.0565	0.0057
					(0.0042)	(0.0004)
	Hedge Funds				0.0026	
					(0.0002)	
Observation Errors						
$\Sigma_{\pi f}$		0.0809	0.0424	0.0289		
		(0.0036)	(0.0019)	(0.0013)		
$\Sigma_{\pi a}$					0.0208	0.0590
					(0.0009)	(0.0026)

Table 5 Continued

Panel B: Risk Aversion

	γ_1				
γ_0	Private	Size	Spending	% Budget	Past Ret
7.48	-0.2594	-0.0700	0.0118	-0.0088	-0.0527
(0.77)	(0.1028)	(0.0652)	(0.0483)	(0.0506)	(0.0463)

Panel C: Alpha Beliefs

		2006	2007	2008	2009	2010	2011	2012
α_0	Private Equity	0.0139	0.0230	0.0311	0.0311	0.0325	0.0370	0.0389
		(0.0024)	(0.0023)	(0.0029)	(0.0024)	(0.0026)	(0.0028)	(0.0030)
	Hedge Funds	0.0029	0.0041	0.0053	0.0054	0.0058	0.0064	0.0066
		(0.0004)	(0.0004)	(0.0006)	(0.0005)	(0.0006)	(0.0006)	(0.0006)
		Private	Size	Spending	% Budget	Past Ret		
α_1	Private Equity	0.0085	0.0256	0.0012	0.0015	-0.0002		
		(0.0012)	(0.0020)	(0.0007)	(0.0007)	(0.0007)		
	Hedge Funds	0.0010	0.0038	0.0002	0.0002	0.0010		
		(0.0002)	(0.0003)	(0.0001)	(0.0001)	(0.0002)		

Table 6: Endowments' Sharpe Ratio and Information Ratio Beliefs

We report estimated Sharpe ratio and information ratio beliefs for endowments. We use the posterior distribution of μ_f, Σ_f, β, Σ_a and α_0 to compute the implied posterior distribution for both measures. These represent the beliefs of the average endowment and therefore the dispersion of the distributions captures parameter uncertainty. Panel A reports the Sharpe ratios for both factor securities and alternative investments. Panel B reports the information ratios for private equity and hedge funds.

Panel A: Sharpe Ratios

	U.S. Stocks	Fixed Income	Foreign Stocks
	0.8369	0.3333	0.6780
	(0.0843)	(0.0347)	(0.0693)

	2006	2007	2008	2009	2010	2011	2012
Private Equity	0.6234	0.6495	0.6726	0.6729	0.6768	0.6897	0.6951
	(0.0721)	(0.0716)	(0.0736)	(0.0733)	(0.0739)	(0.0744)	(0.075)
Hedge funds	0.5847	0.6038	0.6219	0.6234	0.6294	0.6375	0.6414
	(0.0665)	(0.0664)	(0.0676)	(0.0676)	(0.0681)	(0.0684)	(0.0686)

Panel B: Information Ratios

	2006	2007	2008	2009	2010	2011	2012
Private Equity	0.0589	0.0972	0.1311	0.1314	0.1372	0.1562	0.1641
	(0.0108)	(0.0103)	(0.0138)	(0.012)	(0.013)	(0.014)	(0.0151)
Hedge funds	0.0561	0.0809	0.1043	0.1062	0.1139	0.1244	0.1294
	(0.0081)	(0.0079)	(0.0101)	(0.0095)	(0.0103)	(0.0109)	(0.0115)

Table 7: Model-Implied Asset Allocations

The table reports the posterior distribution of the optimal allocations implied by the model. We report the cross-sectional average weight and the cross-sectional standard deviation in parentheses. All numbers are in percentage.

Year	2006	2007	2008	2009	2010	2011	2012	Average
Cash	6.82	6.82	6.39	6.05	4.99	5.42	5.46	5.99
	(2.83)	(2.8)	(2.74)	(2.52)	(2.63)	(2.66)	(2.65)	(2.69)
U.S. stocks	42.18	38.75	35.63	35.68	35.11	33.37	32.67	36.20
	(9.61)	(9.71)	(9.78)	(9.94)	(9.99)	(9.86)	(10.01)	(9.84)
Fixed income	23.19	21.61	20.15	20.15	19.84	19.06	18.73	20.39
	(4.42)	(4.47)	(4.51)	(4.61)	(4.63)	(4.56)	(4.63)	(4.54)
Foreign stocks	13.51	14.31	14.99	14.91	14.93	15.40	15.56	14.80
	(2.62)	(2.62)	(2.6)	(2.51)	(2.56)	(2.56)	(2.57)	(2.58)
Private equity	3.85	5.76	7.44	7.33	7.50	8.55	8.93	7.05
	(5.76)	(5.78)	(5.79)	(5.72)	(5.79)	(5.75)	(5.81)	(5.77)
Hedge funds	10.47	12.75	15.39	15.89	17.63	18.21	18.65	15.57
	(8.39)	(8.49)	(8.55)	(8.75)	(8.8)	(8.66)	(8.8)	(8.63)

Table 8: Trading Off Time-Series Returns and Cross-Sectional Asset Holdings

We report the posterior mean and standard deviation of private equity and hedge fund alphas for different priors, indexed by ν. The case of $\nu = T_f/(T_f + T_i) = 0.19$ corresponds to an uninformative prior, where T_f is the length of the returns data and T_i is the number of cross sections of endowment allocations. The case of $\nu = 0$ corresponds to placing all weight on return time series, and the case of $\nu = 1$ corresponds to using only asset allocations.

		2006	2007	2008	2009	2010	2011	2012
OLS	Private Equity	0.0054						
		(0.0104)						
	Hedge Fund	0.0153						
		(0.0029)						
$\nu = 0.19$	Private Equity	0.0139	0.0230	0.0311	0.0311	0.0325	0.0370	0.0389
		(0.0024)	(0.0023)	(0.0029)	(0.0024)	(0.0026)	(0.0028)	(0.0030)
	Hedge Fund	0.0029	0.0041	0.0053	0.0054	0.0058	0.0064	0.0066
		(0.0004)	(0.0004)	(0.0006)	(0.0005)	(0.0006)	(0.0006)	(0.0006)
$\nu = 0.50$	Private Equity	0.0002	0.0077	0.0152	0.0147	0.0164	0.0198	0.0212
		(0.0028)	(0.0023)	(0.0025)	(0.0023)	(0.0026)	(0.0032)	(0.0033)
	Hedge Fund	0.0010	0.0020	0.0031	0.0030	0.0034	0.0038	0.0040
		(0.0028)	(0.0023)	(0.0025)	(0.0023)	(0.0026)	(0.0032)	(0.0033)
$\nu = 1.00$	Private Equity	0.0093	0.0139	0.0175	0.0173	0.0179	0.0202	0.0206
		(0.0013)	(0.0015)	(0.0016)	(0.0018)	(0.0017)	(0.0017)	(0.0017)
	Hedge Fund	0.0017	0.0023	0.0028	0.0028	0.0030	0.0033	0.0033
		(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)

Table 9: Robustness

Different robustness checks are considered. Instead of using the full sample of returns, we consider the subsample starting from 1970. We use only the allocations of endowments belonging to Ivy league universities plus MIT and Stanford University. We Winsorize funds' characteristics at the 5th and 95th percentiles. We change the indices representing private equities and hedge funds returns to the Cambridge Associates Private Equity Index and the Dow Jones Credit Suisse Hedge Fund Index, respectively. We exclude all endowments with zero weights in alternative assets. We collapse both asset classes into one alternative investment class. We report the posterior mean and standard deviation of average risk aversion γ_0, and the private equity and hedge fund alphas, α_0. All alphas are annualized.

		Alphas α_0					
		2006		2009		2012	
	γ_0	Priv. Eq.	HF	Priv. Eq.	HF	Priv. Eq.	HF
Subsample 1970-2012	11.12	0.0204	0.0043	0.0451	0.0081	0.0551	0.0097
	(0.97)	(0.0046)	(0.0006)	(0.0068)	(0.0009)	(0.0079)	(0.001)
Ivy League Plus	5.47	0.0357	0.0075	0.0512	0.0100	0.0485	0.0090
	(0.64)	(0.0046)	(0.0009)	(0.0058)	(0.0011)	(0.0055)	(0.001)
Winsorized Characteristics	7.04	0.0120	0.0026	0.0276	0.0050	0.0339	0.0060
	(0.63)	(0.0017)	(0.0003)	(0.0026)	(0.0004)	(0.0033)	(0.0005)
Cambridge Private Equity	6.65	0.0061	0.0051	0.0113	0.0092	0.0138	0.0110
	(0.72)	(0.0013)	(0.0008)	(0.0023)	(0.0012)	(0.0027)	(0.0014)
DJ Credit Suisse HF Index	7.14	0.0113	0.0038	0.0244	0.0066	0.0301	0.0078
	(0.65)	(0.0024)	(0.0006)	(0.003)	(0.0009)	(0.0035)	(0.001)
No Alternative Holdings	7.17	0.0082	0.0025	0.0235	0.0049	0.0290	0.0057
	(0.63)	(0.0023)	(0.0004)	(0.0031)	(0.0006)	(0.0036)	(0.0006)
One Alternative Asset Class	5.83	0.0060		0.0111		0.0133	
	(0.77)	(0.0012)		(0.0018)		(0.0022)	

Figure 1: Endowment Asset Allocations

Endowments average asset allocations to U.S. stocks and alternative investments from fiscal year ends 2006 through 2012.

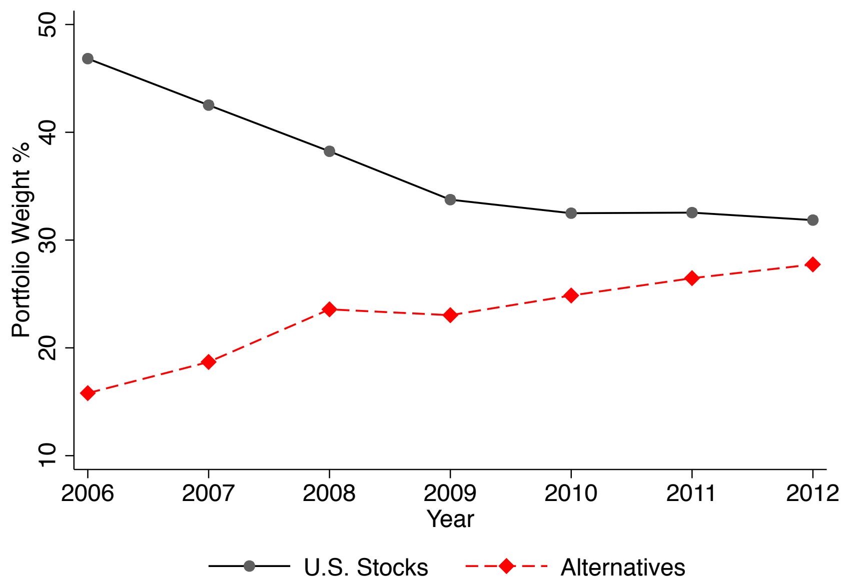
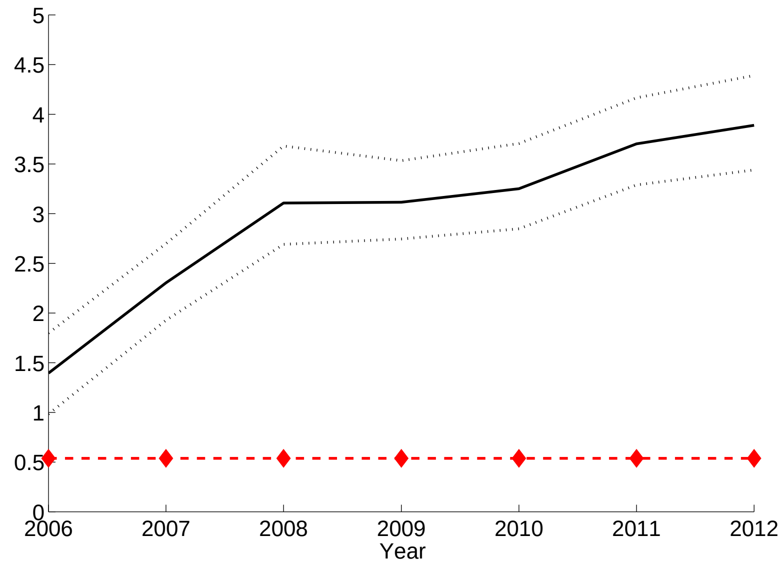


Figure 2: Endowments' Alpha Beliefs for Alternatives

The figure shows the average view on the level of mispricing for private equity (Panel A) and hedge funds (Panel B). We plot the posterior mean of α over time in the solid line, along with 5% and 95% percentiles of the posterior distribution in dotted lines. The dashed line with horizontal triangles represents the estimated alpha from time-series regressions. All numbers are annualized and are in percent.

Panel A: Private Equity



Panel B: Hedge Funds

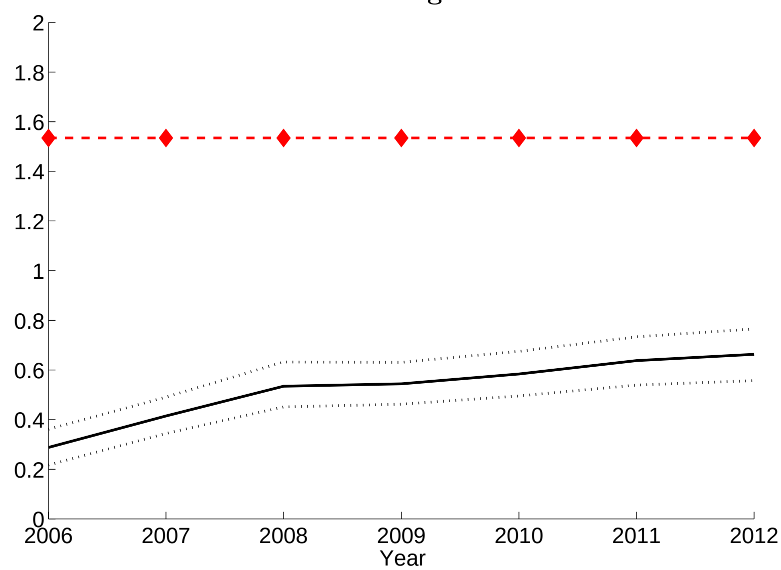


Figure 3: Endowments' Dispersion of Risk Aversion and Alternative Alphas

The figure plots the model-implied cross-sectional distribution of the risk aversion coefficient (top graph), the view on private equity alpha (middle graph), and the view on hedge fund alpha (bottom graph).

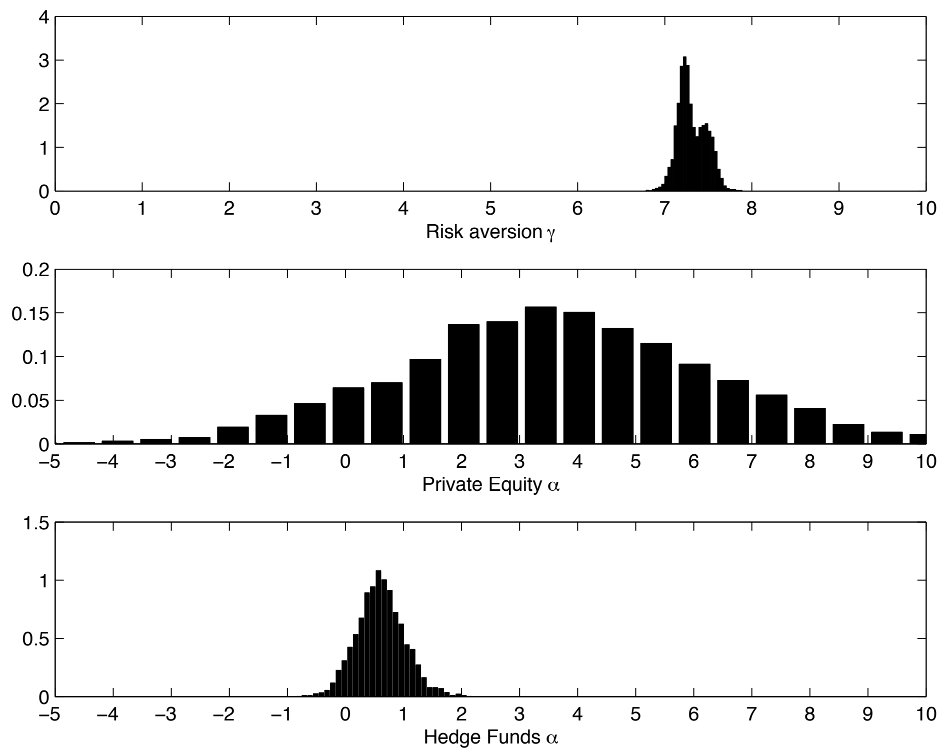
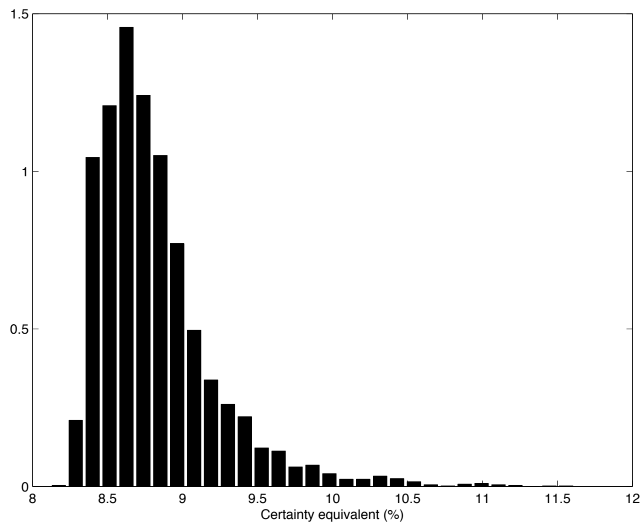


Figure 4: Endowments' Certainty Equivalent Returns

Using the posterior distributions of the parameters, we compute for each endowment-year the certainty equivalent of the portfolio. Panel A shows the distribution of endowments' certainty equivalent returns where the unit of observation is endowment-year. Panel B shows the cross-sectional distribution over the sample period. The black line represents the mean, while the red dotted lines are one standard deviations from the mean. Returns are in percent.

Panel A: Distribution of Certainty Equivalent Returns



Panel B: Cross-Sectional Certainty Equivalent Distribution Over Time

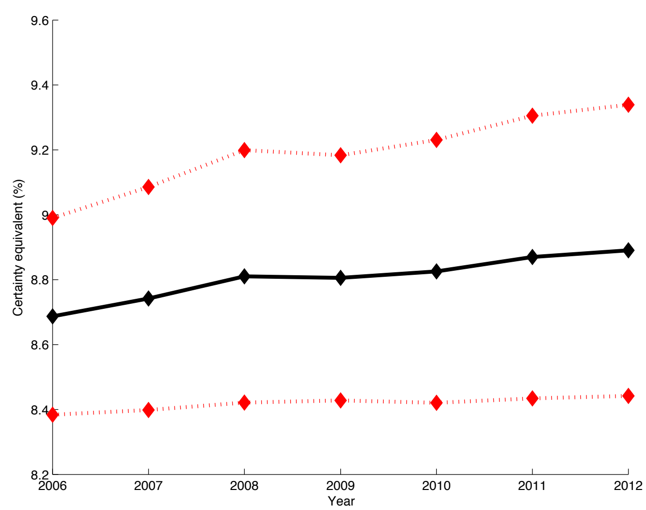
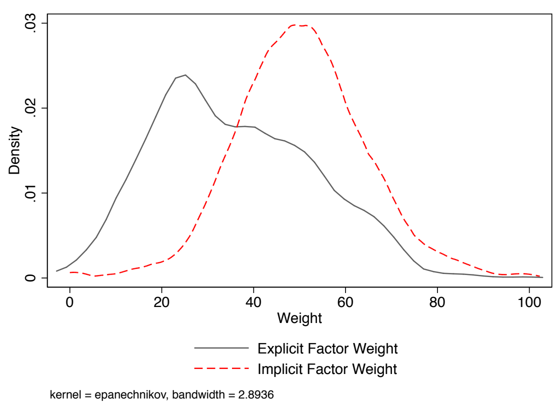


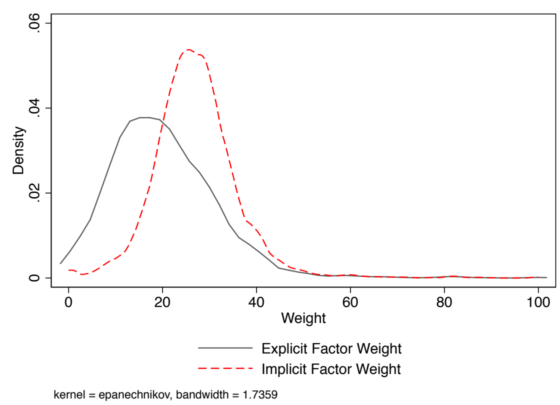
Figure 5: Endowments' Explicit and Implicit Weights on Factor Securities.

Using the estimated factor loadings of alternative investments, β, and the observed asset allocations, we compute the implicit weight on factor securities, $\tilde{\pi}_f = \pi_f + \beta^\top \pi_a$ for each endowment. Figures show kernel estimations of the (pooled) cross-sectional distribution of the explicit (observed) factor weights (solid black) and the implicit factor weight (dashed red).

Panel A: U.S. Stocks



Panel B: Fixed Income



Panel C: Foreign Stocks

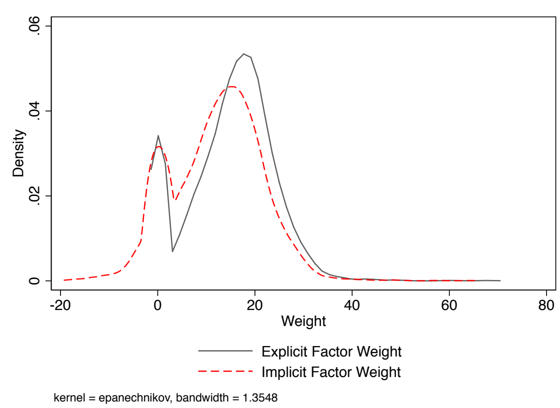
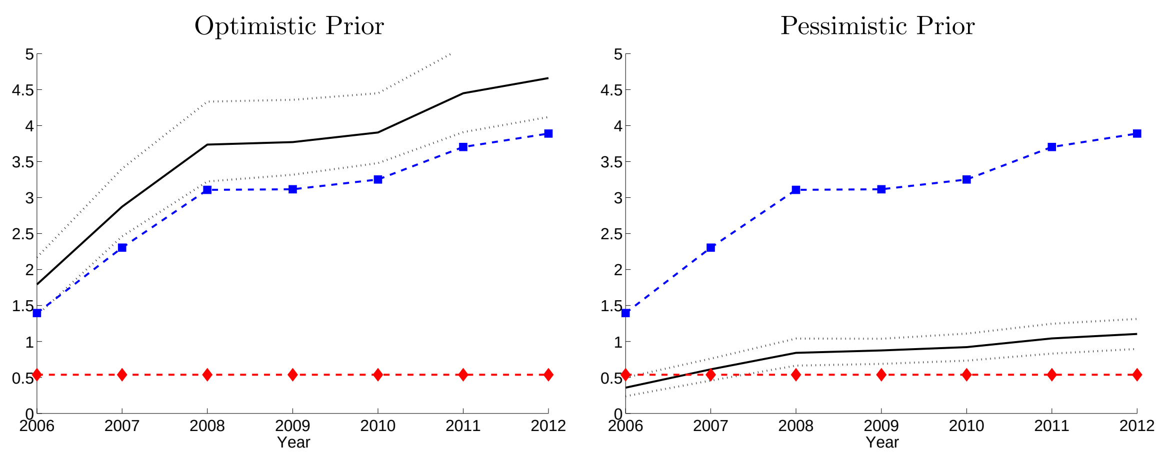


Figure 6: Alpha Beliefs Under Optimistic and Pessimistic Priors

We compute α_0 posterior distribution under two informative prior distributions. For the optimistic case, we assume a normal prior with an annualized mean of 4% and a standard deviation of 2%. For the pessimistic case, we assume a normal prior with a mean of -4% and a standard deviation of 2%. The solid line corresponds to the posterior distribution's mean, while the dotted lines are the distributions' 5th and 95th percentiles. The dashed lines linked by squares represent the posterior average from using a non-informative prior, which is the same as Figure 3.

Panel A: Private Equity



Panel B: Hedge Funds